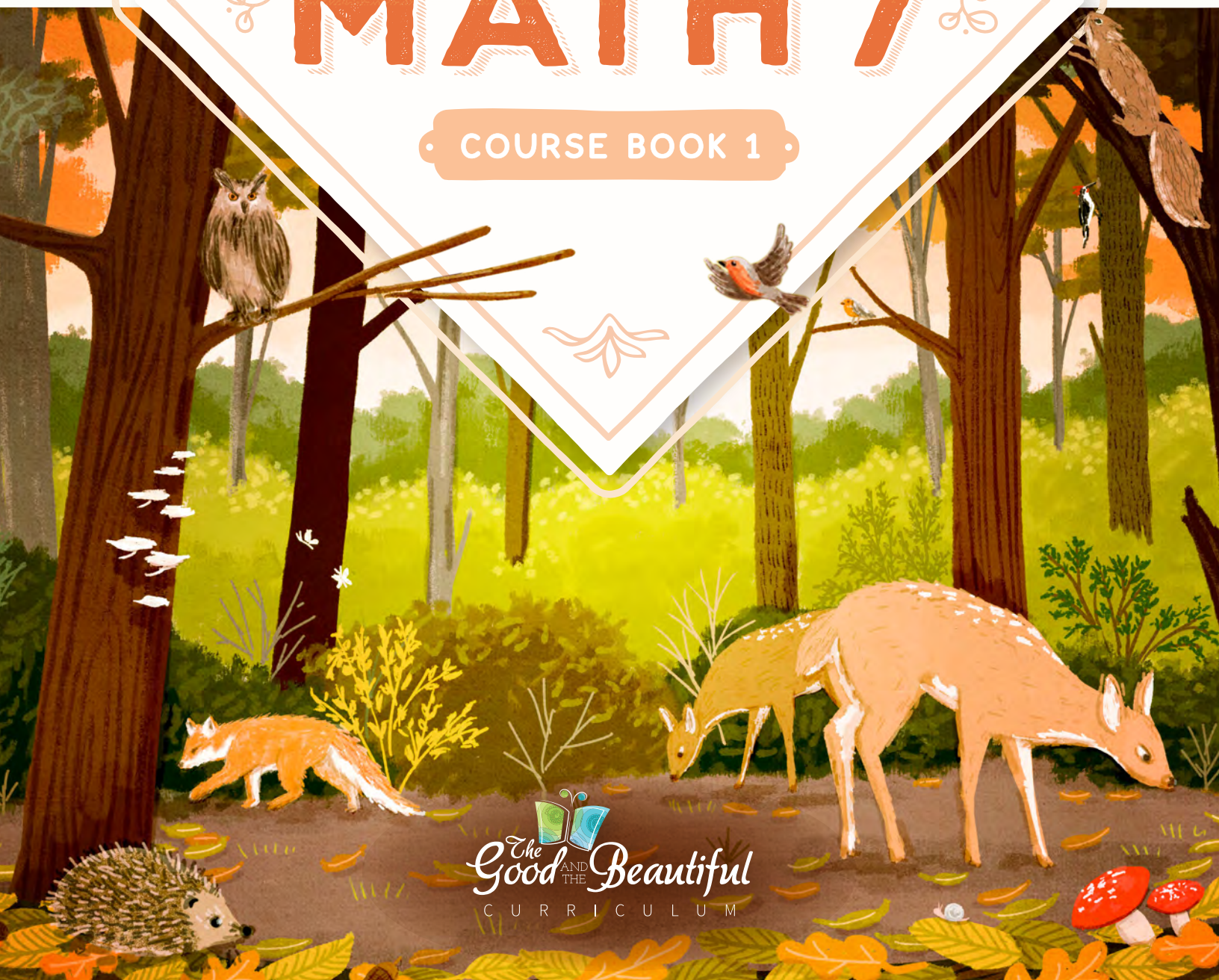
LESSONS
1–30

UNIT
1

Simply Good and Beautiful

MATH 7

COURSE BOOK 1



The Good and the Beautiful
CURRICULUM

COURSE BOOK 1

TABLE OF CONTENTS

About the Course iv

Frequently Asked Questions v

Lesson Topics vi

UNIT 1 OVERVIEW 1

Lesson 1: Writing Decimals, Estimating, and Rounding 2

Lesson 2: Upside Down Division and Prime Factorization 7

Lesson 3: Simplifying Fractions with Prime Factors 13

Lesson 4: Multi-Digit Division 19

Lesson 5: Converting Between Fractions and Decimals 24

Lesson 6: Adding and Subtracting Integers 29

Lesson 7: Multiplying and Dividing Integers 35

Lesson 8: Multiplying and Dividing Fractions 41

Lesson 9: Complex Fractions 46

Lesson 10: Adding and Subtracting Fractions 51

Lesson 11: Adding and Subtracting Decimals 56

Lesson 12: Multiplying and Dividing Decimals 62

Lesson 13: Positive Exponents 67

Lesson 14: Negative Exponents 72

Lesson 15: Logic Lesson 1 77

Lesson 16: Properties of Real Numbers 80

Lesson 17: Expanded Notation with Exponents 85

Lesson 18: Scientific Notation 90

Lesson 19: Operations with Numbers in Scientific Notation 95

Lesson 20: Absolute Value and Coordinate Planes 100

Lesson 21: Order of Operations: Part 1 105

Lesson 22: Order of Operations: Part 2 110

Lesson 23: Simplifying Expressions 115

Lesson 24: Evaluating Expressions 122

Lesson 25: Writing Expressions 126

Lesson 26: Writing Equations 132

Lesson 27: Solving One-Step Equations 138

Lesson 28: Unit 1 Review 144

Lesson 29: Unit 1 Assessment 149

Lesson 30: Enrichment: Sequences and Series 154

Reference Chart 159

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About the Course

SUPPLIES NEEDED

- *Simply Good and Beautiful Math 7 Course Books 1, 2, 3,* and *4*
- *Simply Good and Beautiful Math 7 Answers and Solutions*
- *Simply Good and Beautiful Math Scratch Pad* or other scratch paper
- Device to access videos
- Scientific calculator
- 2 standard dice
- Colored pencils and/or crayons
- Highlighter and/or marker
- Tape or glue
- Ruler
- Protractor
- Compass
- Scissors
- String
- Coin
- Paper clip

COURSE OVERVIEW

Math 7 consists of Course Books 1, 2, 3, and 4. There are 120 total lessons divided into four units. Each unit ends with a unit review, assessment, and enrichment activity. The course is designed to be independently completed by the student, but the parent/teacher can choose to be as involved in the lessons as he or she would like.

GETTING STARTED

Simply open the first course book. The student may choose to watch the video lesson or just read the lesson overview if he or she feels confident in the lesson topic. Please note that videos may contain material not included in the written lesson overview. After completing the video and/or lesson overview, the student should complete the lesson practice and review sections.

The parent/teacher should check the student's work daily and provide immediate help and feedback. Students who struggle with the lesson practice should be encouraged to review the lesson overview or video for help.
Note: If printing at home, print pages at actual size.

LESSON DETAILS

Most lessons consist of a warm-up, video lesson, lesson overview, practice, and review.

WARM-UP: An activity that applies to the lesson topic or that reviews mental math skills.

VIDEO LESSON: Detailed teaching and interactive, guided practice of the lesson topic. Videos are about 12–15 minutes in length.

The Good and Beautiful Homeschool app can be used to access and watch the lesson videos. Use the QR code below to access app download information.

Alternatively, the videos can be accessed at goodandbeautiful.com/Math7.

LESSON OVERVIEW: A concise written lesson on the topic.

PRACTICE: Problems dedicated to the lesson topic.

REVIEW: Daily review of topics from previous lessons.

A Reference Chart can be found at the back of each book.

IMPORTANT SAFETY NOTICE

Adult supervision is recommended for all activities as some require sharp objects, heat sources, or small objects that could be a choking hazard for young children.

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Frequently Asked Questions

How many lessons should my student do each week?

- There are 120 lessons in the course. If the student completes four lessons per week, he or she will complete the course in a standard school year with typical breaks for vacation or sickness.

How long do lessons take?

- The average time to complete a lesson is 50–60 minutes. This includes time to watch the video and complete the course book sections.

What if my student does not do well on an assessment?

Each assessment question has a lesson number indicating where the content was first introduced. If your student misses an assessment question, he or she is encouraged to do one or more of the following:

- Reread the corresponding lesson overview.
- Rewatch the corresponding video.
- Complete the extra practice worksheet for the corresponding lesson (available for purchase).
- Rework the problem given the answer. It can be helpful to know the answer when reworking a problem so mistakes can be found.

Do you include any specific doctrine?

- No, the goal of our curriculum is not to teach doctrines specific to any particular Christian denomination but to teach general principles, such as honesty, hard work, and kindness. All Bible references in our curriculum are from the King James Version.

Does my student have to watch the videos?

- The videos contain the bulk of the teaching and are highly recommended. However, if your student feels confident in the topic being taught, he or she can skip the video and read the lesson overview instead. A student who struggles with the lesson practice should be encouraged to go back and watch the video.
- Some families prefer to have the parent/teacher facilitate the lesson using the lesson overview rather than have the student watch the video lesson independently.

Is Math 7 completed independently by the student?

- Yes, Math 7 is designed for your student to complete independently, though at times the student may need parent/teacher assistance to understand a concept. The parent/teacher will need to check the student's work and should do so on a daily basis when possible, providing immediate feedback.

Is Math 7 a spiral or mastery program?

- Math 7 is a spiral course, constantly reviewing concepts your student has learned to ensure understanding and retention of information.

What if there isn't room to complete the work?

- Math 7 is designed to give students room to work in their course book. At times, additional paper may be needed. Students should always keep scratch paper on hand while completing the lessons. The *Simply Good and Beautiful Math Scratch Pad* is available for purchase.

Is a calculator used in Math 7?



- This course is designed to be completed with a scientific calculator on hand for specific problems. Problems that allow the use of a calculator are marked with the calculator icon shown to the left. Any brand of scientific calculator is acceptable. Please note that calculators may vary, and your student is encouraged to read the manual for the specific calculator to understand how it functions.

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Lesson Topics

UNIT 1

1 Writing Decimals, Estimating, and Rounding
2 Upside Down Division and Prime Factorization
3 Simplifying Fractions with Prime Factors
4 Multi-Digit Division
5 Converting Between Fractions and Decimals
6 Adding and Subtracting Integers
7 Multiplying and Dividing Integers
8 Multiplying and Dividing Fractions
9 Complex Fractions
10 Adding and Subtracting Fractions
11 Adding and Subtracting Decimals
12 Multiplying and Dividing Decimals
13 Positive Exponents
14 Negative Exponents
15 Logic Lesson 1
16 Properties of Real Numbers
17 Expanded Notation with Exponents
18 Scientific Notation
19 Operations with Numbers in Scientific Notation
20 Absolute Value and Coordinate Planes
21 Order of Operations: Part 1
22 Order of Operations: Part 2
23 Simplifying Expressions
24 Evaluating Expressions
25 Writing Expressions
26 Writing Equations
27 Solving One-Step Equations
28 Unit 1 Review
29 Unit 1 Assessment
30 Enrichment: Sequences and Series

UNIT 2

31 Set Notation
32 Evaluating Square Roots
33 Solving Two-Step Equations
34 Square Roots and Cube Roots
35 Multi-Step Equations with Negative Coefficients
36 Solving Equations Review
37 Solving for a Variable in Terms of Other Variables
38 Solving and Graphing One-Step Inequalities
39 Solving and Graphing Multi-Step Inequalities
40 Fractions of a Group
41 Ratios and Proportions
42 Solving Ratio Problems: Part 1
43 Solving Ratio Problems: Part 2
44 Rounding Fractions and Mixed Numbers
45 Logic Lesson 2
46 Percentages
47 Percent Increase
48 Percent Decrease
49 Simple Interest
50 Compound Interest
51 Identifying Unit Rates
52 Proportions Within Similar Triangles
53 Metric and US Customary Units
54 Unit Conversions
55 Converting Square Units
56 Operations with Mixed Measures
57 Mixed Review
58 Unit 2 Review
59 Unit 2 Assessment
60 Enrichment: Graph Theory

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UNIT 3

61 Scale Drawings
62 Direct Proportions
63 Inverse Proportions
64 Graphs of Direct Proportions
65 Graphing Using a T-Chart
66 Slope of a Line
67 Slope-Intercept Form
68 Graphing Linear Equations
69 Functions
70 Graphing Functions
71 Triangles
72 Transformations
73 Constructing Angles
74 Constructing Triangles
75 Logic Lesson 3
76 Polygon Diagonals and Angles
77 Finding Polygon Angle Measures
78 Angle Relationships
79 Parallel Lines and Transversals
80 Missing Angles in a Circle
81 Pythagorean Theorem
82 Perimeter of Polygons
83 Area of Polygons
84 Area and Circumference of Circles
85 Composite Figures
86 Inscribed Shapes
87 Mixed Review
88 Unit 3 Review
89 Unit 3 Assessment
90 Enrichment: Circumference and Diameter

UNIT 4

91 Scale Factor with Area
92 Arcs and Sectors
93 Geometric Solids
94 Surface Area of Prisms and Pyramids
95 Surface Area of Cylinders, Cones, and Spheres
96 Surface Area of Composite Solids
97 Volume of Prisms and Cylinders
98 Volume of Other Geometric Solids
99 Polynomials
100 Multiplying Polynomials
101 Simplifying Rational Expressions
102 Factoring Polynomials
103 Populations and Sampling Methods
104 Data Displays: Part 1
105 Logic Lesson 4
106 Measures of Central Tendency
107 Interpreting Measures of Central Tendency
108 Data Displays: Part 2
109 Scatter Plots
110 Interpreting Graphs
111 Simple Probability
112 Types of Events
113 Sample Space
114 Compound Probability
115 Probability Simulation
116 Unit 4 Review
117 Course Review
118 Course Assessment
119 Enrichment: Patterns with Divisibility
120 Fun with Graphing

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UNIT 1 OVERVIEW

LESSONS 1–30

CONCEPTS COVERED

- Adding and subtracting decimals
- Adding and subtracting fractions
- Adding and subtracting integers
- Applying reasoning to determine validity of answers
- Combining like terms
- Complex fractions
- Converting between fractions and decimals
- Converting between standard form and scientific notation
- Coordinate planes
- Equations with negative numbers
- Estimating and rounding
- Evaluating expressions
- Evaluating expressions with positive exponents
- Evaluating integers raised to negative exponents
- Expanded notation with exponents
- Expressions, constants, and coefficients
- Greatest common factor
- Identifying and writing equations
- Identifying solutions to equations
- Least common multiple
- Multiplying and dividing decimals
- Multiplying and dividing fractions
- Multiplying and dividing integers
- Multiplying and dividing numbers in scientific notation
- Operations with signed fractions and decimals
- Opposites and absolute value
- Prime factorization
- Prime factorization to simplify fractions
- Properties of real numbers
- Simplifying division problems
- Simplifying expressions using the order of operations
- Solving and checking one-step equations
- Terminating and repeating decimals
- Upside down division
- Using absolute value to find horizontal and vertical distances on coordinate planes
- Using calculators
- Writing expressions
- Writing large numbers with digits and words
- Zero as an exponent and base

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UNIT 1 | LESSON 1

Writing Decimals, Estimating, and Rounding

SUPPLIES: colored pencils

WARM-UP

Multiply or divide.

a. $45 \div 15$ b. $16 \bullet 4$ c. $56 \div 8$

LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Notes section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

App Video

VIDEO NOTES

VOCABULARY

- ***Terminating decimal:*** a decimal number with a _____ number of _____ after the decimal point

  Examples of terminating decimals:

- ***Repeating decimal:*** a decimal number with _____ or _____ digits after the _____ _____ that repeat forever

  Examples of repeating decimals:

Estimate: _____

Exact decimal answer: $4.8 \bullet 0.513 =$ _____

Rounded answer: $4.8 \bullet 0.513 \approx$ _____

Is the answer reasonable based on the estimate? _____

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LESSON OVERVIEW

Terminating and Repeating Decimals

Numbers used in real-world situations are often decimal numbers. There are different types of decimal numbers. Two kinds of decimal numbers are terminating decimals and repeating decimals.

Terminating Decimal

A ***terminating decimal*** is a decimal number with a limited number of digits after the decimal point.

Examples:

0.25, 6.1283, 4.8

Repeating Decimal

A ***repeating decimal*** is a decimal number with one or more digits after the decimal point that repeat forever.

Examples:

0.333..., 5.1919..., 0.111...

Repeating decimals are written with a bar over the repeating digit(s). 0.333... can be written as $0.\overline{3}$ with a bar over the repeating 3.

The following division has been completed using a calculator.

KEY INFORMATION

Calculators often round the last digit of a repeating decimal.

$7 \div 9 = 0.7777777778...$

The digit 7 repeats, so $7 \div 9 = 0.\overline{7}$

$1 \div 12 = 0.08333333333...$

The digit 3 repeats, so $1 \div 12 = 0.08\overline{3}$

Look for a pattern in the digits after the decimal point.

$27 \div 53 = 0.509433962264150943396226415...$

The digits 5094339622641 repeat, so $27 \div 53 = 0.\overline{5094339622641}$

Estimating and Rounding

It can be useful to estimate answers before performing calculations. This helps determine if a mistake was made during calculation and if the answer is reasonable.

Example 1: Estimate the answer to $13 \div 7$. Then divide and round the quotient to the nearest ten thousandth.

14 is a whole number close to 13 that is divisible by 7. → **Estimate:** $14 \div 7 = 2$

Use a calculator to divide: $13 \div 7 = 1.857142857142...$

Round to the nearest ten thousandth: $13 \div 7 \approx 1.8571$

This is close to the estimated answer of 2.

KEY INFORMATION

The symbol ≈ means "approximately equal to."

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Example 2: Estimate the answer to 3.81 • 9.25. Then multiply and round the product to the nearest thousandth.

3.81 is close to 4, and 9.25 is close to 9. → **Estimate:** 4 • 9 = 36
These are whole numbers that are easy to multiply.

Use a calculator to multiply: 3.81 • 9.25 = 35.2425

Round to the nearest thousandth: 3.81 • 9.25 ≈ 35.243

This is close to the estimated answer of 36.

Example 3: Estimate 6.98 • 4.43. Then multiply and round the product to the nearest tenth.

6.98 is close to 7, and 4.43 can be rounded down to 4. → **Estimate:** 7 • 4 = 28

Use a calculator to multiply: 6.98 • 4.43 = 30.9214

Round to the nearest tenth: 6.98 • 4.43 ≈ 30.9

This is close to the estimated answer of 28.

Example 4: Divide and round the quotient to the nearest hundredth.

a. **0.125 ÷ 0.8**

Use a calculator to divide: 0.125 ÷ 0.8 = 0.15625

Round to the nearest hundredth: 0.125 ÷ 0.8 ≈ 0.16

b. **37 ÷ 22**

Use a calculator to divide: 37 ÷ 22 = 1.68181818181...

Round to the nearest hundredth: 37 ÷ 22 ≈ 1.68

PRACTICE

A calculator may be used for problems with this symbol.

1. Estimate each quotient by using nearby numbers that divide evenly.

a. 19 ÷ 5 _____ b. 47 ÷ 15 _____

2. Determine whether the answer to each problem in the table is a terminating or a repeating decimal and place a check mark in the appropriate column.

Problem	Answer Terminates	Answer Repeats
8.52 • 4.09		
103 ÷ 3		
39 ÷ 3		
68.6868 • 4.44		
56 ÷ 3		

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3. Round each answer to the nearest ten thousandth.

a. $33 \div 13$ _____

b. $4.56 \bullet 2.6398$ _____

c. $8.623 \bullet 5.01$ _____

4. Write each answer as an exact decimal using a bar.

a. $98 \div 15$ _____

b. $65 \div 12$ _____

c. $134 \div 11$ _____

5. a. Estimate the quotient of $34 \div 5$ by rounding 34 to the closest multiple of 5.

_____

b. Will the quotient be greater than or less than the estimate?

Circle one: *greater than* / *less than*

6. Five friends evenly split a dinner bill that totaled $34. How much did each person pay?

$_____

7. Tammy is cutting a piece of poster board that is 34 cm wide into five equal strips. How wide will each strip be?

_____ cm

8. A total of 34 students are going on a field trip. Each car can hold five students. How many cars are needed?

_____ cars

9. Match each problem with its exact decimal answer by drawing the same pattern and/or using the same color. An example is given.

$65 \div 6$	$11.\overline{5}$	12.6
$63 \div 5$	$115 \div 10$	$38 \div 3$
$10.8\overline{3}$	$1266 \div 100$	$10.\overline{83}$
$11.\overline{56}$	12.66	12.7
$254 \div 20$	11.5	$1145 \div 99$
$104 \div 9$	$12.\overline{6}$	$1073 \div 99$



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REVIEW

Math Facts Review

In each triangle the bottom two numbers multiplied together equal the top number. Fill in the missing number on each triangle. Then write the fact family below each triangle. An example is given. If a box has a letter next to it, write the letter on the line above the corresponding answer at the bottom of the page to solve the riddle.

Example: top: 32; bottom: 4, A 8

4	×	8	=	32
8	×	4	=	32
32	÷	4	=	8
32	÷	8	=	4

1. top: O ☐; bottom: 5, 6

	×		=	
	×		=	
	÷		=	
	÷		=	

2. top: 24 R; bottom: ☐ T, 6

	×		=	
	×		=	
	÷		=	
	÷		=	

3. top: 54 S; bottom: 6, E ☐

	×		=	
	×		=	
	÷		=	
	÷		=	

4. top: 63 I; bottom: ☐ D, 9

	×		=	
	×		=	
	÷		=	
	÷		=	

5. top: ☐ M; bottom: 12, 3

	×		=	
	×		=	
	÷		=	
	÷		=	

6. top: ☐ H; bottom: 11, 2

	×		=	
	×		=	
	÷		=	
	÷		=	

7. top: ☐ C; bottom: 9, 8

	×		=	
	×		=	
	÷		=	
	÷		=	

8. top: ☐ N; bottom: 7, 10

	×		=	
	×		=	
	÷		=	
	÷		=	

Why didn't the dime roll down the mountain with the nickel?

Because ___ ___   ___ A ___   ___ ___ ___ ___   ___ ___ ___ ___ ___!

63 4   22 8 7   36 30 24 9   72 9 70 4 54

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Five-in-a-Row

5. Roll two standard dice and add the values. Find ONE square containing the sum (orange numbers). Complete the problem in that square, and then color in the square. Continue until you have five squares in a row colored. The five squares in a row can be connected horizontally, vertically, or diagonally.

12 $-20 \div (-4)$	**2** $-3 \bullet 30$	**7** $-150 \div 15$	**4** $48 \div 8 \div (-3)$	**5** $90 \bullet (-7)$
7 $-1 \bullet 76$	**8** $-64 \div 8$	**9** $-84 \div (-7)$	**10** $-9 \bullet (-12)$	**11** $8 \bullet 7 \bullet (-1)$
6 $-5 \bullet 0$	**2** $10 \bullet (-1)$	**3** $-40 \div (-4)$	**4** $-72 \div 9 \div 2$	**5** $99 \div (-9)$
7 $32 \div (-4)$	**8** $42 \div 7$	**9** $12 \bullet 11$	**10** $-18 \div (-18)$	**11** $11 \bullet (-11)$
12 $54 \div (-6)$	**6** $-7 \bullet 7$	**3** $25 \bullet 6$	**4** $0 \bullet (-23)$	**5** $-66 \div 6$



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3. Add. Write answers as mixed numbers.

a. $3\frac{1}{3}+5\frac{5}{18}$ _____

b. $-3\frac{1}{3}+\left(-5\frac{5}{18}\right)$ _____

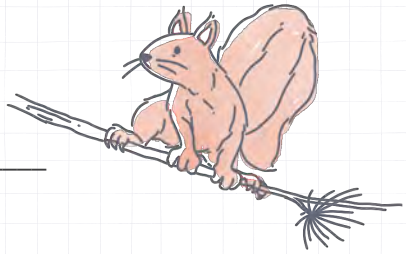
REMEMBER:

Change mixed numbers to improper fractions before adding or subtracting.

4. Add.

a. $\frac{21}{25}+\left(-\frac{3}{5}\right)$ _____

b. $\frac{3}{5}+\left(-\frac{21}{25}\right)$ _____



Aunt Laurie's Banana Chocolate Chip Cookies

Cream together in a large mixing bowl:

____ a. cup(s) sugar

____ b. cup(s) butter

____ c. egg(s)

Beat into mixture:

____ d. cup(s) mashed banana

____ e. tablespoon(s) baking powder

____ f. teaspoon(s) salt

Mix in:

____ g. cup(s) flour

Stir in:

____ h. cup(s) chocolate chips

Place balls of dough on a baking sheet. Bake at 350 °F for 8–10 minutes.

Be careful when using an oven, and ask an adult for help as needed.

5. Add or subtract. Write answers in the blanks to complete the recipe for banana chocolate chip cookies.

a. $10\frac{1}{6}-8\frac{2}{3}$ _____

b. $-3\frac{8}{9}+4\frac{24}{27}$ _____

c. $\frac{10}{4}+\frac{8}{16}$ _____

d. $6\frac{2}{5}+\left(-4\frac{9}{10}\right)$ _____

e. $\frac{1}{6}-\left(-1\frac{1}{3}\right)$ _____

f. $-\frac{3}{10}+\frac{11}{20}$ _____

g. $7\frac{5}{6}-\frac{49}{12}$ _____

h. $-\frac{3}{7}+2\frac{3}{7}$ _____

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Positive Exponents

SUPPLIES: ruler or straightedge

WARM-UP

Evaluate each expression.

a. $3+3+3+3$ b. $3 \cdot 3 \cdot 3 \cdot 3$ c. $5+5+5$ d. $5 \cdot 5 \cdot 5$

LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Notes section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

App Video

VIDEO NOTES

4^3 _____ _____ _____

$4^3 =$ ____ • ____ • ____ = ____

When multiplying an even amount of negative numbers, the product will be _____.

When multiplying an odd amount of negative numbers, the product will be _____.

$(-2.5)^3 =$ _____ • _____ • _____ = _____

Exponents and Zero

- Any nonzero base raised to the power of _____ is _____.
- Zero to any nonzero power is _____.
- Zero to the power of zero is _____.

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LESSON OVERVIEW

The expression 4^3 is called a power and is read as "4 to the third power." It has an exponent and a base. The ***base*** is the number that is multiplied by itself when using an exponent. The ***exponent*** is a number showing how many times to multiply the base number by itself. Note: The word "power" can be used to refer to the whole expression (e.g., a power of 4), or it can refer to the exponent itself (e.g., 4 to the power of 3).

4^3 ← exponent
base

4^3 can be written in factored form as $4 \cdot 4 \cdot 4$. It can also be evaluated. $4^3 = 4 \cdot 4 \cdot 4 = 64$

Exponents can be used to write prime factorizations. Here is the prime factorization of 56 in exponential form. $56 = 2 \cdot 2 \cdot 2 \cdot 7 = 2^3 \cdot 7$

A prime factorization with exponents can be written in factored form and evaluated. Here is a prime factorization evaluated. $2^4 \cdot 3 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 = 48$

REMEMBER: A number raised to the second power is referred to as *squared*. A number raised to the third power is referred to as *cubed*.

Fractional and Decimal Bases

The base can be any number or expression. To evaluate a power, write the expression in factored form and multiply.

Example 1: $\left(\frac{1}{2}\right)^4$

$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

$= \frac{1}{16}$

Example 2: $\left(\frac{9}{5}\right)^2$

$= \frac{9}{5} \cdot \frac{9}{5}$

$= \frac{81}{25} = 3\frac{6}{25}$

Example 3: $\left(1\frac{2}{3}\right)^3$

$= \left(\frac{5}{3}\right)^3$

$= \frac{5}{3} \cdot \frac{5}{3} \cdot \frac{5}{3}$

$= \frac{125}{27} = 4\frac{17}{27}$

Example 4: $\frac{5^2}{11}$ ← Only the numerator is squared.

$= \frac{5 \cdot 5}{11}$

$= \frac{25}{11} = 2\frac{3}{11}$

Example 5: $\frac{7}{3^3}$ ← Only the denominator is cubed.

$= \frac{7}{3 \cdot 3 \cdot 3}$

$\frac{7}{27}$

Example 6: 6.5^3

$= 6.5 \cdot 6.5 \cdot 6.5$

$= 274.625$

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2. Rewrite each power as a fraction with a positive exponent. Do not evaluate the power.

a. 5^{-13} _____ b. 8^{-9} _____

c. 11^{-10} _____ d. $(-6)^{-3}$ _____

e. -12^{-10} _____ f. $(-7)^{-4}$ _____

3. Evaluate. Write each answer as a simplified fraction.

a. $(-2)^{-6}$ _____ b. 12^{-2} _____

c. -4^{-4} _____ d. $(-10)^{-3}$ _____

4. Rewrite each power of 10 as a decimal number.

a. 10^{-1} _____

b. 10^{-2} _____

c. 10^{-8} _____

5. Rewrite each decimal number as a power of 10.

a. 0.00001 _____

b. 0.001 _____

c. 0.0000000001 _____

6. Preposterous Preferences

Read the two options and decide which one you would prefer. Rewrite the value in that box in the form specified. For extra practice, complete both problems in each part.

a. Swim in a pool of . . .

Vanilla pudding	Root beer
$\frac{1}{10000}$ → power of 10	$\frac{1}{100000}$ → power of 10

Vanilla pudding: _____ OR Root beer: _____

b. Sleep in a bed of . . .

Sand	Harmless snakes
5^{-3} → simplified fraction	2^{-5} → simplified fraction

Sand: _____ OR Harmless snakes: _____

c. Eat a raw . . .

Potato	Onion
0.1 → power of 10	0.000001 → power of 10

Potato: _____ OR Onion: _____

d. Have four . . .

Arms	Legs
$\frac{1}{27}$ → power of 3	$\frac{1}{36}$ → power of 6

Arms: _____ OR Legs: _____

e. Time travel to the . . .

Past	Future
7^{-3} → simplified fraction	4^{-3} → simplified fraction

Past: _____ OR Future: _____

f. Have a . . .

Nose like a platypus	Neck like a giraffe
$\frac{1}{16}$ → power of 2	$\frac{1}{81}$ → power of 3

Nose like a platypus: _____
OR Neck like a giraffe: _____

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Logic Lesson 1

Nine-Hole Golf

A standard golf course has 18 holes, but a nine-hole game can also be played. Solve the following nine puzzles that each involve the number 9. There is no video or review for this lesson.

Hole 1

Use the digits 1 through 9 exactly once each to create an addition problem with a sum of 99.

✦ Hint: Use one 2-digit addend.

____ + ____ + ____ + ____ +

____ + ____ + ____ + ____ = 99

Hole 2

Using the digits 1 through 9 exactly once, place three numbers in each hole so that the sum of the numbers in each hole is the same.

Sum of each hole: ____

Hole 3

If you multiply Cory's age by $5\frac{1}{2}$, you get his great-grandfather's age, which is 99. How old is Cory?

____ years old

Hole 4

Fill in each of the four holes with a different digit between 1 and 9 to create a true statement. The fraction must result in an exact decimal that ends in the tenths place.

$\frac{\square}{\square\square}$ = 0.$\square$

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Hole 9

Four families went golfing. Each family went to a different course and played a different number of holes. Use the clues to figure out which course and what number of holes each family played.

- Hint: Once you know something for certain, put a ✓ in that box and fill in the rest of the row and column of that 4 × 4 box with Xs. You may need to go through the clues more than once.
- Hint: Golfers don't have to play an entire course. For example, if a course has 18 holes, golfers can play *fewer* than 18 holes but not *more* than 18 holes.

		Course				Number of Holes Played			
		Greenfield Golf	Putter's Paradise	Par for the Course	The Cart Club	9	18	27	36
Family	Stewart								
	Lin								
	Miller								
	O'Brien								
Number of Holes Played	9								
	18								
	27								
	36								

1. The Miller family played more holes than the Stewart family, but fewer holes than the O'Brien family.
2. Putter's Paradise has 27 holes. This means the family who played this course played 9, 18, or 27 holes.
3. The family who played at Par for the Course played 27 holes.
4. The O'Brien family played more than 18 holes.
5. The Lin family played four times the number of holes as the Stewart family.
6. The Miller family did not play at Putter's Paradise.
7. Greenfield Golf has 18 holes. This means the family who played this course played 9 or 18 holes.
8. The Lin family played at The Cart Club.
9. The Miller family played a course that does not start with the letter P.

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6. Multiply or divide. Perform operations inside parentheses first.

a. $(2 \cdot 5) \cdot 6$ ____ b. $2 \cdot (5 \cdot 6)$ ____ c. $(24 \div 12) \div 2$ ____ d. $24 \div (12 \div 2)$ ____

7. For Parts A, B, and C, find the number that makes a true statement. For Parts D, E, and F, simplify the expression. Then locate all the boxes containing each answer on the grid and shade in the sections in that box according to the design listed next to the problem.

a.  $11 \cdot 8 = ____ \cdot 11$

b.  $\frac{1}{23} \cdot ____ = 1$

c.  $28 + ____ = 0$

d.  $7(40 - 1)$

e.  $7 \cdot (4 \cdot 3)$

f.  $3(80 + 1)$

a. _____ b. _____ c. _____

d. _____ e. _____ f. _____

84	273	8	23	23	273	8	243
273	84	243	8	273	84	243	8
243	8	273	84	243	8	273	84
–28	243	84	273	8	243	84	–28
–28	273	8	243	84	273	8	–28
273	84	243	8	273	84	243	8
243	8	273	84	243	8	273	84
8	243	84	23	23	243	84	273

REVIEW

1. Evaluate each power. L13, L14

a. 11^{-2} ____ b. 8^{-2} ____ c. $\left(\frac{5}{9}\right)^2$ ____ d. $\left(\frac{3}{4}\right)^4$ ____

2. Add or subtract. L10

a. $2\frac{3}{16} - 7\frac{5}{6}$ _____ b. $9\frac{1}{27} + \left(-8\frac{2}{9}\right)$ _____

3. Multiply or divide. L12

a. $15.9 \cdot (-6.1)$ _____

b. $-20 \div (-3.6)$ _____



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d. 50.74

Expanded form: _____

Expanded notation: _____

Expanded notation with exponents: _____

4. **Error Detective**

Look closely at the expanded form, expanded notation, or expanded notation with exponents for each number. Find and highlight the error(s). Then rewrite the part(s) of the expression containing errors correctly on the line.

a. $50265 = 50000 + 2000 + 60 + 5$

Correction(s): _____

b. $9166.3 = 9100 + 60 + 6 + 0.3$

Correction(s): _____

c. $0.97 = (9 \bullet 10^{1}) + (7 \bullet 10^{2})$

Correction(s): _____

d. $0.15 = (1 \bullet 10^{0}) + (5 \bullet 10^{-1})$

Correction(s): _____

e. $26.039 = (2 \bullet 10) + (6 \bullet 1) + (3 \bullet 0.1) + (9 \bullet 0.01)$

Correction(s): _____

f. $7.602 = (7 \bullet 1) + (6 \bullet 0.1) + (0.01) + (2 \bullet 0.001)$

Correction(s): _____

g. $64.78 = (6 \bullet 10^{2}) + (4 \bullet 10^{1}) + (7 \bullet 10^{-1}) + (8 \bullet 10^{-2})$

Correction(s): _____

h. $0.409 = (4^{-1}) + (9^{-3})$

Correction(s): _____

REVIEW

1. Multiply or divide.

a. $7.64 \bullet 10000$ _____

b. $8.23 \div 1000$ _____

2. Use the distributive property to simplify each expression. L16

a. $-6(7x - 9)$ _____

b. $18(-2y - 1)$ _____

3. Evaluate each power. L13

a. $(-5)^{6}$ _____

b. -12^{4} _____

4. Write the prime factorization of each number using exponents. L2, L13

a. 147 _____

b. 216 _____

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3. The approximate populations of three nations in 2020 are given in standard form. Rewrite each number in scientific notation.

   a. China:
   1,440,000,000 _____

   b. United States of America:
   331,000,000 _____

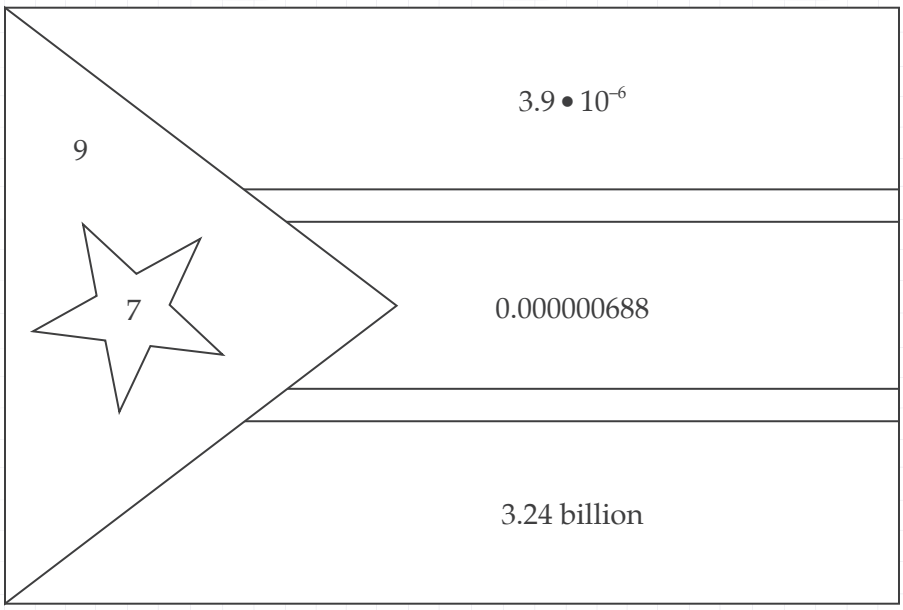
   c. Sweden:
   10,380,000 _____

4. The atomic radius of an iron atom is approximately 1.4×10^{-10} meters. Rewrite this number in standard form.

   _____

5. The atomic radius of a hydrogen atom is approximately 0.000000000023 meters. Rewrite this number in scientific notation.

   _____



6. The Flag of South Sudan

Formally established in 2011, South Sudan is one of the newest countries in the world.

For Parts A and B, determine the value of the missing exponent. For Parts C–H, complete the problem given. If the answer appears on the flag, color that section of the flag the color specified in the problem.

a. Yellow: The population of South Sudan in 2020 was about 11,200,000. In scientific notation this number is $1.12 \times 10^{?}$.

   _____

b. Blue: In 2020 the population of Africa was about 1,300,000,000. In scientific notation this number is $1.3 \times 10^{?}$.

   _____

c. Red: Write 6.88×10^{-7} in standard form.

   _____

d. Purple: Write 6.88×10^{-9} in standard form.

   _____

e. Green: Write 3,240,000,000 using a combination of numbers and words.

   _____

f. White: Write 32,400,000 using a combination of numbers and words.

   _____

g. Orange: Write 0.000039 in scientific notation.

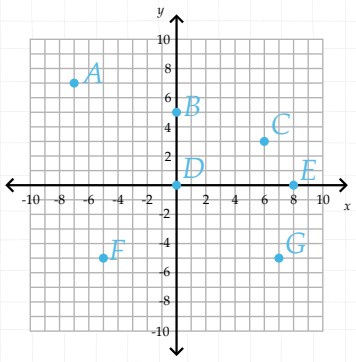
   _____

h. Black: Write 0.0000039 in scientific notation.

   _____

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3. Write the coordinates of the points shown below.



A: _____ *B*: _____ *C*: _____

D: _____ *E*: _____ *F*: _____

G: _____

4. Find the absolute value of each number.

a. 5 ____ b. –7 ____ c. $-\frac{5}{9}$ ____

d. 12.7 ____ e. 0 ____

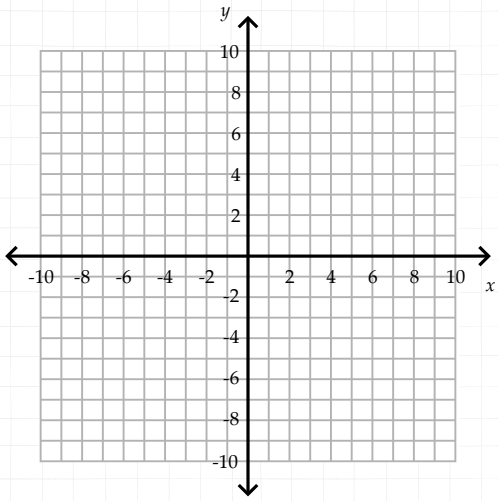
5. Using absolute values, find the distance between the following pairs of points.

a. $(1,4)$ and $(7,4)$ _____

b. $(-2,3)$ and $(-2,-7)$ _____

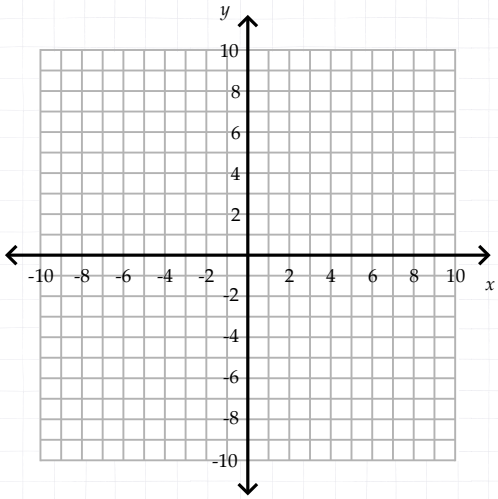
c. $(0,0)$ and $(-5,0)$ _____

6. Plot the pairs of points in Problem 5 using a different color for each pair. Draw a line connecting each pair. Verify that the distance between each pair matches your answers for Problem 5.



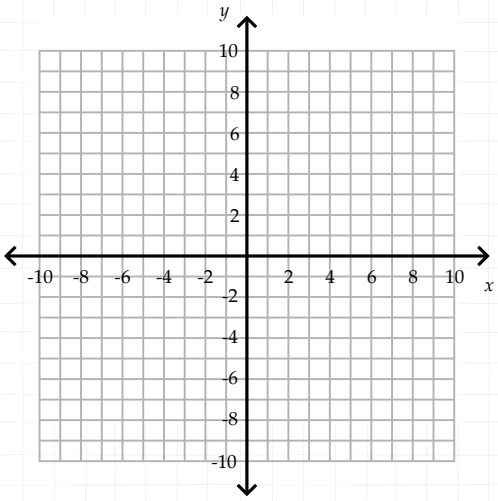
7. Write the coordinates of two points that are on the *x*-axis and are each a distance of four units from the origin. Plot the points on the coordinate plane.

_____ and _____

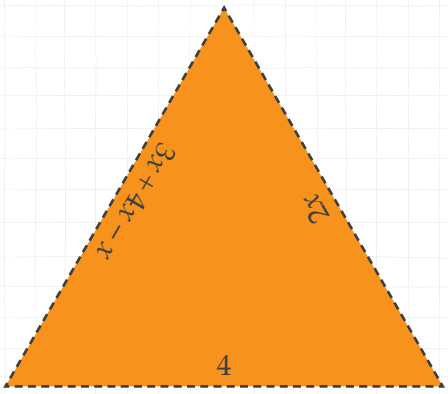


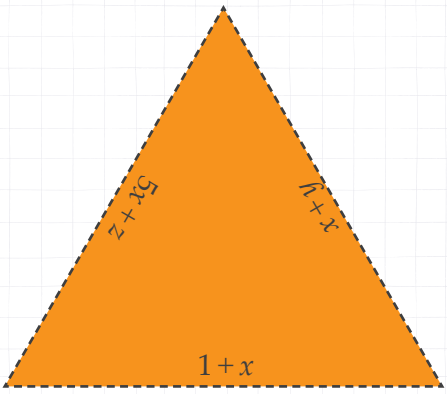
8. Write the coordinates of a point located in Quadrant II that is a distance of three units from $(0,5)$. Plot the point on the coordinate plane.

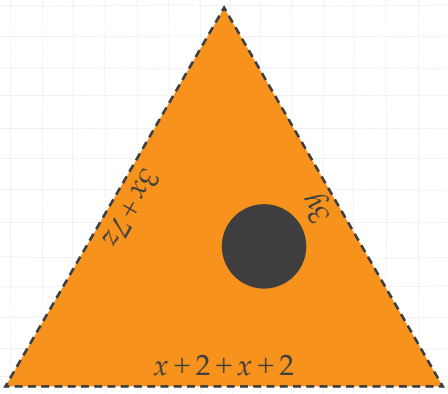
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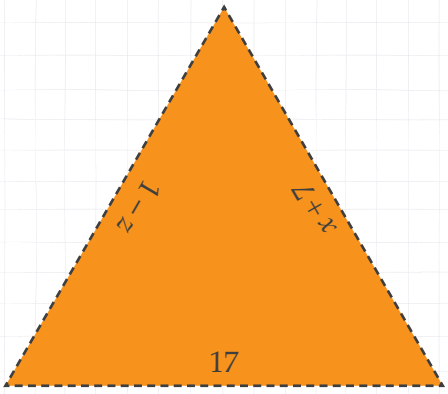


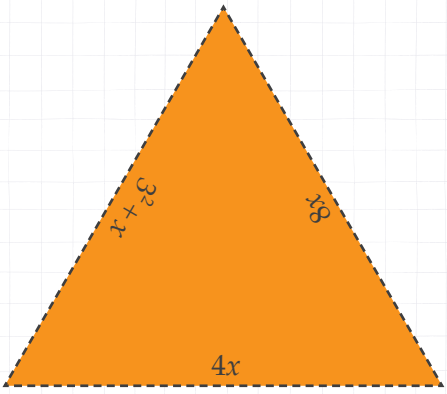
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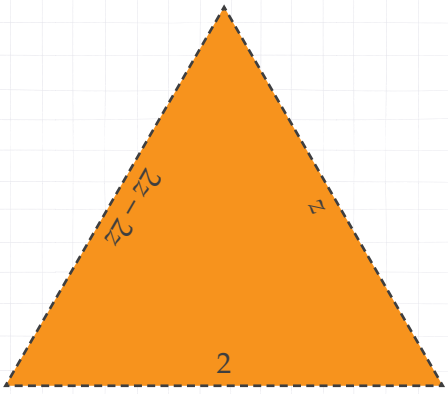


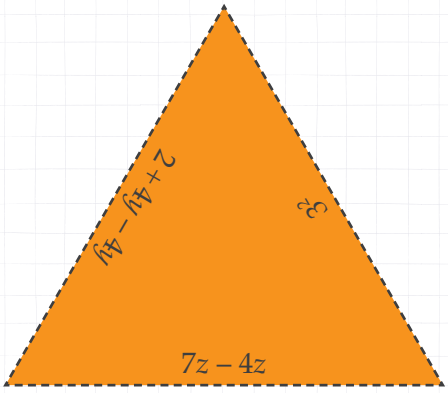


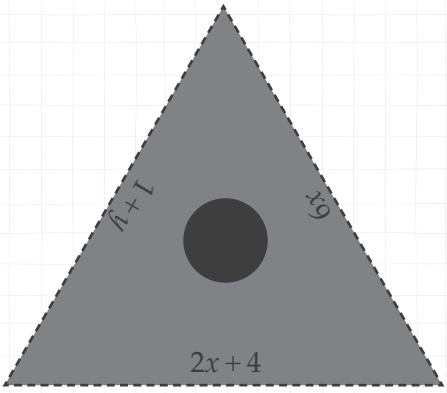


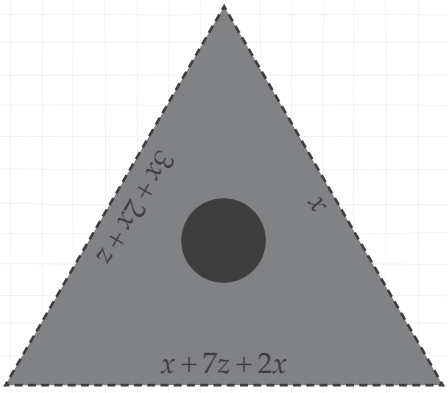


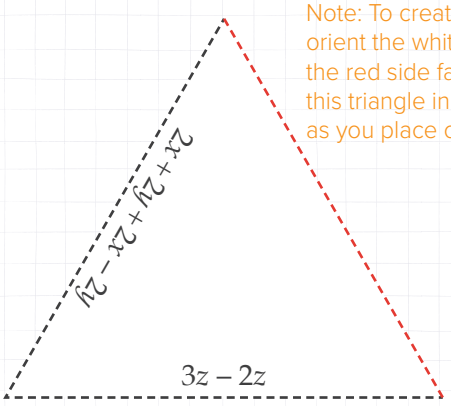




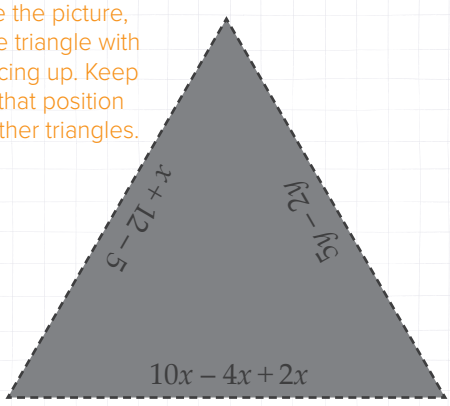


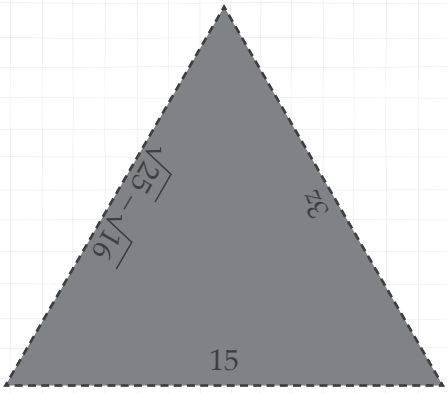






Note: To create the picture, orient the white triangle with the red side facing up. Keep this triangle in that position as you place other triangles.





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Evaluating Expressions

WARM-UP

Evaluate the expressions.

a. $(-8)^2 - 6^2$

b. $\sqrt[3]{27}$

LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Notes section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

App Video

VIDEO NOTES

1. $\dfrac{6x^2}{xy-8}$ $x =$ _____ $y =$ _____

2. $= \dfrac{6(\quad)^2}{(\quad)(\quad)-8}$

3. $= \dfrac{6(\quad)}{-8}$

4. $=$ ____

5. $= -$ ____

6. $= -$ ____

$f + \dfrac{60}{d} - \sqrt{e}$

$d =$ _____, $e =$ _____, $f =$ _____

$8\left(n - \sqrt{p}\right) \div m$

$m =$ _____, $n =$ _____, $p =$ _____

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LESSON OVERVIEW

To evaluate an expression means to find the value of an expression when the variable is replaced by a given number. When evaluating expressions, substitute the value of the variable or variables into the expression. Parentheses may be needed around the substituted value(s) when substituting a negative number or to indicate multiplication. Use the order of operations when evaluating expressions. The number substituted into an expression is referred to as the *input*, and the resulting value is referred to as the *output*.

Here are three different expressions evaluated when $a=-2$. In each expression, a is replaced with -2, and then the expression is evaluated. Notice that parentheses are needed around -2 because it is a negative number.

$$3a+5$$
$$3(-2)+5$$
$$=-6+5$$
$$=-1$$

$$a^3-7$$
$$(-2)^3-7$$
$$=-8-7$$
$$=-15$$

$$16+a\div 2$$
$$16+(-2)\div 2$$
$$=16-1$$
$$=15$$

The three expressions below have more than one variable. Each expression is evaluated when $r=8$, $s=-5$, and $t=9$. Parentheses are only used for substitutions when needed.

$$2\left(s-\sqrt{t}\right)\div r$$
$$2\left(-5-\sqrt{9}\right)\div 8$$
$$=2(-5-3)\div 8$$
$$=2(-8)\div 8$$
$$=-16\div 8$$
$$=-2$$

$$10t-r\bullet s$$
$$10(9)-8\bullet(-5)$$
$$=90-(-40)$$
$$=130$$

$$t(6+s)+\sqrt[3]{r}$$
$$9(6+(-5))+\sqrt[3]{8}$$
$$=9(1)+\sqrt[3]{8}$$
$$=9(1)+2$$
$$=9+2$$
$$=11$$

Example 1:

Evaluate $\dfrac{6x^2}{xy-8}$ when $x=5$ and $y=-1$.

$$\frac{6(5)^2}{(5)(-1)-8}$$
$$=\frac{6(25)}{-5-8}$$
$$=\frac{150}{-13}$$
$$=-11\frac{7}{13}$$

KEY INFORMATION

The value substituted into an expression is called the input. The result, once the expression is simplified, is called the output.

Example 2:

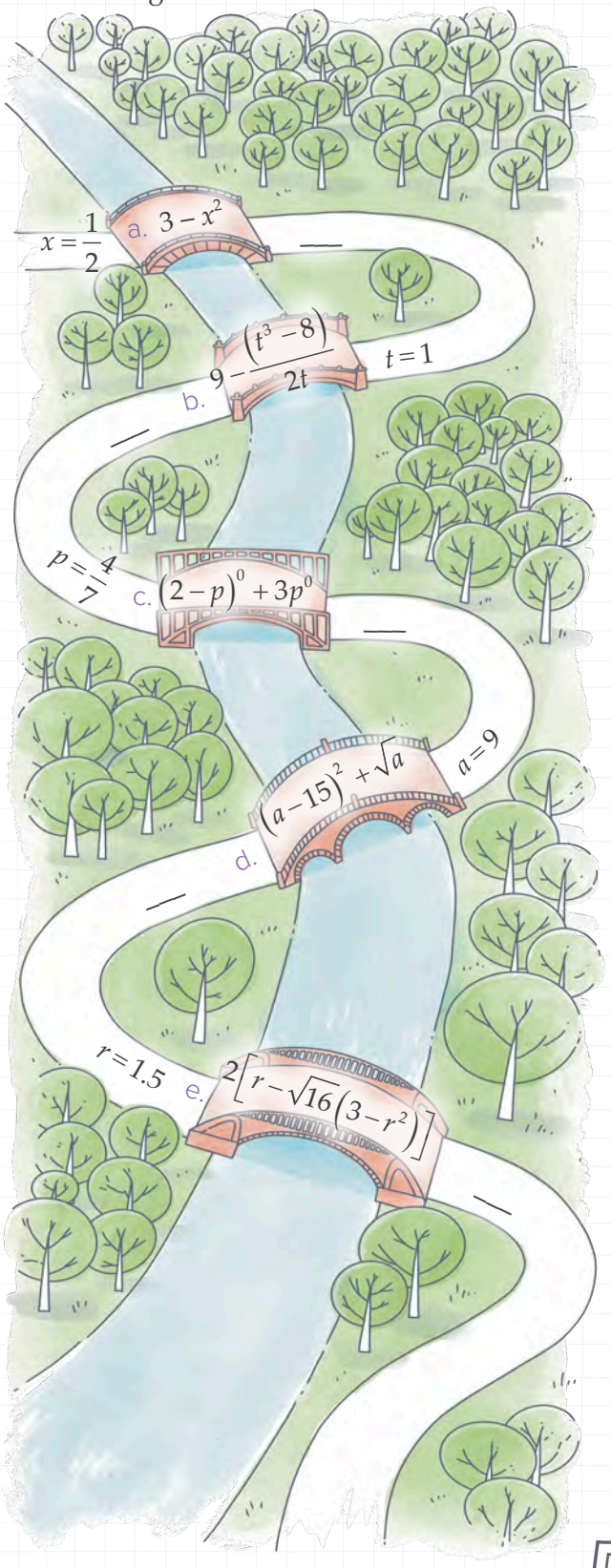
Evaluate $\dfrac{6x^2}{xy-8}$ when $x=\dfrac{1}{3}$ and $y=12$.

$$\frac{6\left(\frac{1}{3}\right)^2}{\left(\frac{1}{3}\right)(12)-8}$$
$$=\frac{6\left(\frac{1}{9}\right)}{4-8}$$
$$=\frac{\frac{2}{3}}{-4}$$
$$=-\frac{1}{6}$$

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PRACTICE

1. Each bridge has an expression at the top. Using the input values at the start of each bridge, determine the output and write it at the end of each bridge.



2. Fill in the table by evaluating the expressions using the given values.

✦ Hint: Replace *y* and *z* in each expression with the values given at the top of each column.

	$y = 0$, $z = 0$	$y = 5$, $z = 0$	$y = -10$, $z = -3$
$\frac{y-5}{y+5} + z$			
$yz - (1 + y)$			
$(y - z)(y + z)$			

3. A part of each expression is missing. Fill in the box with a number so that the given input and output values work in the expression. An example is given.

a. Expression: $b + \boxed{4} - 2(b + 1)$

Input: $b = 5$ Output: -3

Substitute 5 in place of b. $5 + \boxed{} - 2(5 + 1)$

The missing value must be 4 for the expression to equal -3.

b. Expression: $\frac{s(-1 + \boxed{})}{2}$

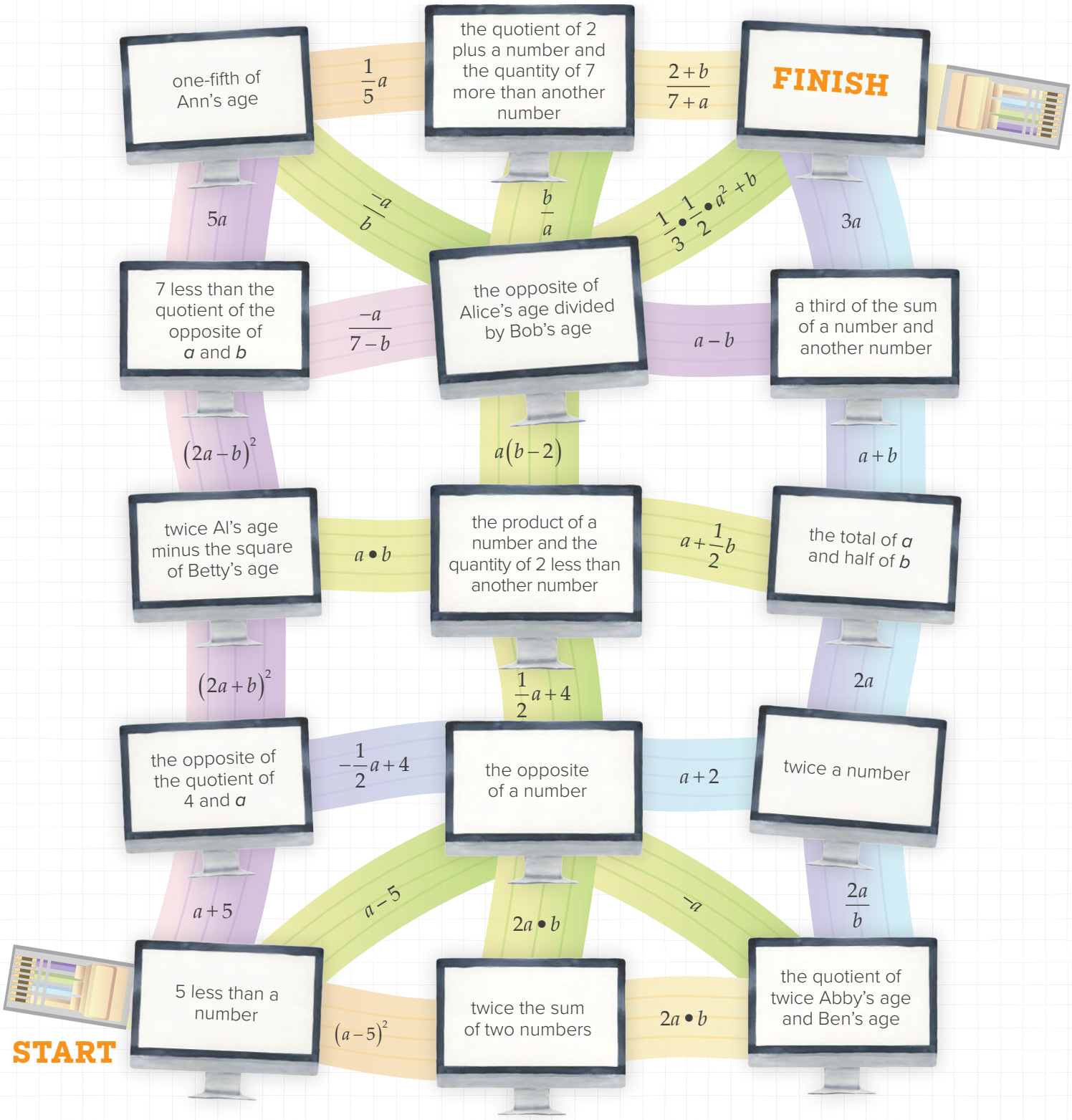
Input: $s = 12$ Output: 66

c. Expression: $2t^2 + \sqrt[3]{\boxed{}} - r$

Input: $t = -2$, $r = 5$ Output: 5

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6. Complete the maze by following the paths with the correct expression for the situation given.



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Complete this Unit Review to prepare for the Unit Assessment. There is no video, lesson, or practice. Because Unit Reviews include practice for an entire unit, they may take longer than regular lessons, and students may decide to take two days to finish.

UNIT 1 | LESSON 28

Unit 1 Review

ESCAPE ROOM

To get out of the Escape Room, you must solve the following riddle:

How can the number four be half of five?

Complete these 10 challenges to find the keys you need to solve the riddle and escape!

Challenge #1: Decimals and Fractions

Lessons 1, 4–5

1. There are 241 keys in a basket. It takes you 3.7412 seconds to try each key in the keyhole. Estimate how long it will take you to try all the keys. Then use a calculator to find the exact time and round the answer to the nearest thousandth.

   Estimate: _____

   Rounded: _____



2. Divide. Write the answer as an exact decimal.

   $632 \div 48$

   _____



3. Fill in the table to convert between fractions and decimals.

Fraction	Decimal
	0.6
$\frac{2}{3}$	
$4\frac{5}{8}$	



Challenge #2: Prime Numbers and Simplifying

Lessons 2–3

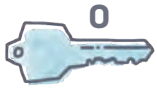
Your next challenge is to simplify fractions with impossibly large numerators and denominators. Simplify using upside down division and prime factorization to make the impossible possible!

4. $\frac{420}{600}$

   _____

5. $\frac{248}{1240}$

   _____



Challenge #3: Adding and Subtracting Signed Numbers

Lessons 6, 10–11

6. Fill in each table by evaluating the given expressions. The value that does NOT match the others in each table gets the key! Be careful: you may have to convert between fractions and decimals to figure out which one is different.



Expression	Value
$4-10+2+7-1$	
$-3+5-7+4-2$	
$-1+4-2-(-1)$	

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Riddle Solution

Use all capital letters when filling in the riddle!

___ ($\frac{3}{25}$) ___ (901.629) ___ (21)   ___ (-6.5) ___ ($\frac{1}{5}$) ___ ($2\frac{1}{2}$) ___ ($-9\frac{3}{25}$) ___ ($0.\overline{6}$)   ___ ($0.\overline{6}$) ___ (-3.5) ___ ($2\frac{1}{2}$) ___ (21) ___ (-6.5) ___ ($-9\frac{3}{25}$) ___ (5×10^2)

___ (-6.5) ___ (21) P ___ (-6.5) ___ (21) ___ (-3) ___ (21) ___ ($0.\overline{6}$) ___ ($\frac{3}{25}$) ___ (4.23×10^{-8}) ___ ($0.\overline{6}$) G   ___ ($\frac{3}{25}$) ___ (901.629) ___ (21)

___ ($0.\overline{6}$) ___ (-3.5) ___ ($2\frac{1}{2}$) ___ (3) ___ (21) ___ (-6.5)   ___ (12) ___ ($\frac{1}{5}$) ___ (-3.5) ___ (-6.5)   ___ (4.23×10^{-8}) ___ (-3)   ___ (4.23×10^{-8}) ___ (-4.08),

___ ($\frac{1}{2}$) ___ (901.629) ___ (4.23×10^{-8}) ___ (2) ___ (901.629)   ___ (4.23×10^{-8}) ___ (-3)   ___ (21) X ___ ($-9\frac{3}{25}$) ___ (2) ___ ($\frac{3}{25}$) ___ (5×10^2) ___ ($13.1\overline{6}$)

___ (901.629) ___ ($-9\frac{3}{25}$) ___ (5×10^2) ___ (12)   ___ ($\frac{1}{5}$) ___ (12)   ___ ($\frac{3}{25}$) ___ (901.629) ___ (21)   ___ (5×10^2) ___ (21) ___ ($\frac{3}{25}$) ___ ($\frac{3}{25}$) ___ (21) ___ (-6.5) ___ (-3)

___ (4.23×10^{-8}) ___ ($0.\overline{6}$)   ___ ($\frac{3}{25}$) ___ (901.629) ___ (21)   ___ ($\frac{1}{2}$) ___ ($\frac{1}{5}$) ___ (-6.5) D   ___ (12) ___ (4.23×10^{-8}) ___ (-4.08) ___ (21).

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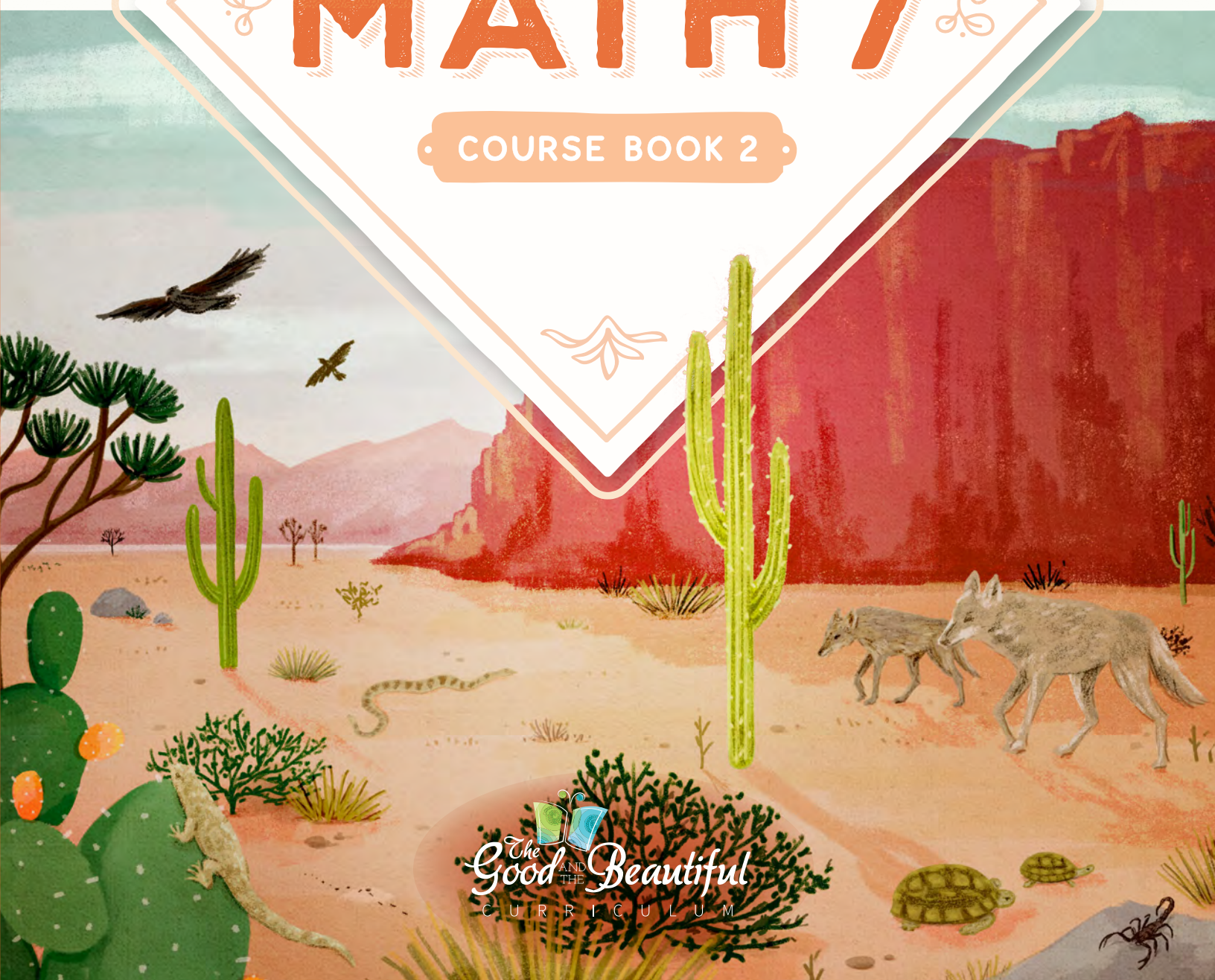
LESSONS
31–60

UNIT
2

Simply Good and Beautiful

MATH 7

COURSE BOOK 2



The Good and the Beautiful
CURRICULUM

COURSE BOOK 2

TABLE OF CONTENTS

UNIT 2 OVERVIEW 1

Lesson 31: Set Notation 2
Lesson 32: Evaluating Square Roots 8
Lesson 33: Solving Two-Step Equations 14
Lesson 34: Square Roots and Cube Roots 20
Lesson 35: Multi-Step Equations with Negative Coefficients 25
Lesson 36: Solving Equations Review 31
Lesson 37: Solving for a Variable in Terms of Other Variables 34
Lesson 38: Solving and Graphing One-Step Inequalities 39
Lesson 39: Solving and Graphing Multi-Step Inequalities 44
Lesson 40: Fractions of a Group 50
Lesson 41: Ratios and Proportions 56
Lesson 42: Solving Ratio Problems: Part 1 61
Lesson 43: Solving Ratio Problems: Part 2 66
Lesson 44: Rounding Fractions and Mixed Numbers 72
Lesson 45: Logic Lesson 2 78
Lesson 46: Percentages 82
Lesson 47: Percent Increase 87
Lesson 48: Percent Decrease 93
Lesson 49: Simple Interest 99
Lesson 50: Compound Interest 104
Lesson 51: Identifying Unit Rates 109
Lesson 52: Proportions Within Similar Triangles 115
Lesson 53: Metric and US Customary Units 121
Lesson 54: Unit Conversions 128
Lesson 55: Converting Square Units 134
Lesson 56: Operations with Mixed Measures 140
Lesson 57: Mixed Review 146
Lesson 58: Unit 2 Review 150
Lesson 59: Unit 2 Assessment 155
Lesson 60: Enrichment: Graph Theory 160

Reference Chart 167

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UNIT 2 OVERVIEW

LESSONS 31–60

CONCEPTS COVERED

- Applying inequalities to real-life scenarios
- Compound interest formula
- Converting percents to decimals
- Converting percents to fractions
- Converting units in the metric system
- Converting units in the US customary system
- Determining if ratios form a proportion
- Evaluating square roots using a calculator
- Finding a percent decrease
- Finding a percent given a whole and a part
- Finding a percent increase
- Finding a whole given a percent and a part
- Finding an original or new amount given a percent decrease
- Finding an original or new amount given a percent increase
- Finding part of a whole given a fraction and the whole
- Finding the fraction given the whole and a part
- Finding the percent of a number
- Finding the whole given a fraction and the part
- Given a part-to-part ratio, finding a missing part or whole
- Given a part-to-whole ratio, finding a missing part or whole
- Graphing inequalities on number lines
- Irrational numbers
- Multiple ways to solve equations
- Natural numbers, whole numbers, integers, rational numbers
- Perfect squares and cubes
- Performing operations with mixed measures
- Plotting irrational numbers on a number line
- Real number system
- Set notation and symbols for set notation
- Simple interest formula
- Solving and checking two-step equations
- Solving equations with negative coefficients
- Solving equations with square and cube roots
- Solving equations with squared and cubed variables
- Solving for a variable in terms of other variables
- Solving for missing sides in congruent triangles
- Solving for missing sides in similar triangles
- Solving multi-step inequalities
- Solving one-step inequalities with negative coefficients
- Solving proportions using cross products
- Solving proportions using equivalent ratios
- Solving two-step equations with exponents and roots
- Total amount of investments
- Unit rates from tables
- Unit rates from word problems
- Using formulas to solve problems
- Using unit multipliers in word problems
- Using unit multipliers to convert between systems of measurement
- Using unit multipliers to convert units of area
- Using unit multipliers to convert within systems of measurement
- Word problems with two-step equations
- Writing and comparing ratios
- Writing ratios and proportions for real-life scenarios
- Writing unit multipliers from any conversion factors

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UNIT 2 | LESSON 31

Set Notation

SUPPLIES: colored pencils

WARM-UP

Draw the correct comparison symbol.

a. $10 \bigcirc |14 - 24|$

b. $14 \bigcirc 7^2$

c. $\sqrt{121} \bigcirc -15$

LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Notes section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

App Video

VIDEO NOTES

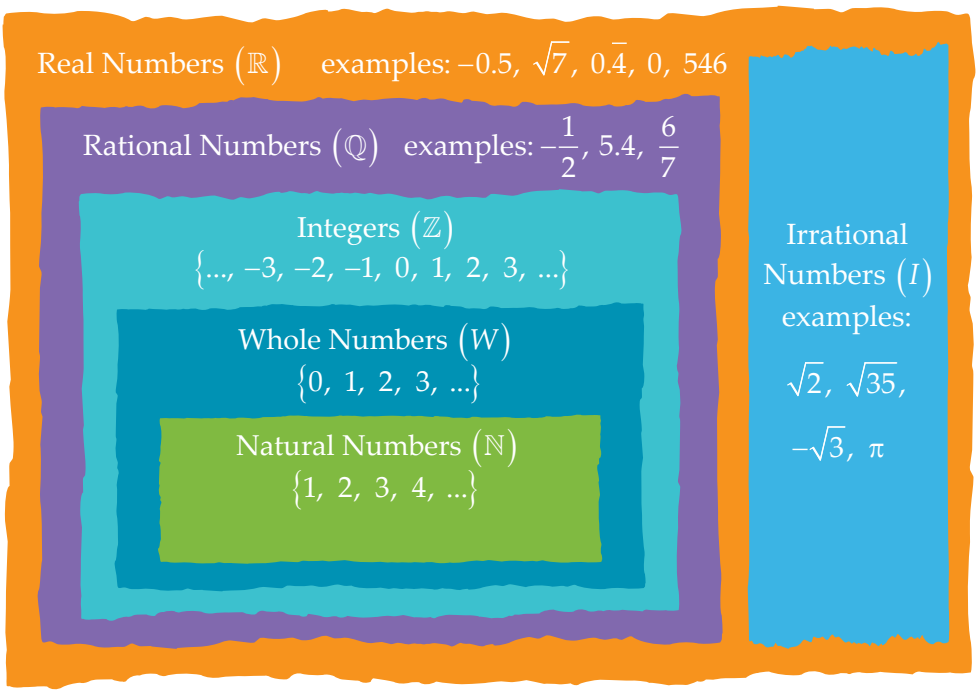
Rational numbers	a group or collection of objects
Real numbers	the set of all rational and irrational numbers
Integer	numbers that can be written as fractions where both the numerator and denominator are integers and the denominator is not zero
Natural numbers	numbers that cannot be written as fractions
Whole numbers	a number with no fractional or decimal part
Set	numbers representing a whole amount (not a fraction or decimal)
Irrational numbers	the numbers we say when we count

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$\subseteq$	"is an element of" (or "is in")
$\in$	"intersect"
$\notin$	"is a subset of"
$\cap$	"the empty set"
$\cup$	"is not an element of" (or "is not in")
$\varnothing$	"the complement of Set A"
A'	"union"

LESSON OVERVIEW

Organizing large groups of things can make them easier to understand. Numbers can be organized into sets. A ***set*** is a group or collection of objects. Number sets describe different characteristics of numbers. Below is a diagram showing the relationships between different number sets.

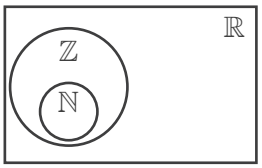
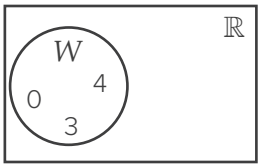
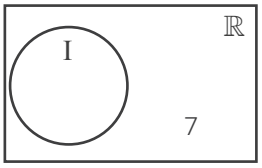
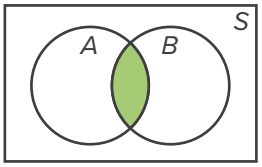
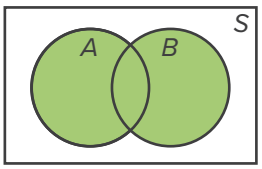
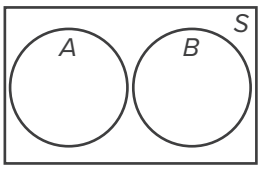
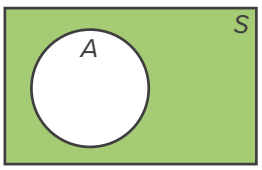


Real numbers are the set of all rational and irrational numbers. Any point on the number line is a real number.

Irrational numbers are numbers that cannot be written as fractions. All real numbers that are not rational numbers are irrational numbers. When written in decimal form, the decimal expansion of an irrational number is infinite (it does not end or repeat). Any irrational number must be rounded to be written as a decimal.

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This chart shows symbols used for set notation. Objects in a set are referred to as ***elements***.

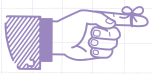
Symbol	Meaning	Examples in Words	Example in Symbols	Illustration
$\subseteq$	"is a subset of" A subset is a set that is entirely within another set.	The set of natural numbers is a subset of the set of integers.	$\mathbb{N} \subseteq \mathbb{Z}$	
$\in$	"is an element of" or "is in"	4 is an element of the set of whole numbers.	$4 \in W$	
$\notin$	"is not an element of" or "is not in"	7 is not in the set of irrational numbers.	$7 \notin I$	
$\cap$	"intersect" The intersection of sets contains all the numbers the sets have in common.	A intersect B or A and B or The intersection of A and B.	$A \cap B$	
$\cup$	"union" The union of sets contains everything included in both sets.	A union B or A or B or The union of A and B.	$A \cup B$	
$\varnothing$	"the empty set" The empty set is a set that contains no elements.	If two sets have no elements in common, the intersection of sets is the empty set.	$A \cap B = \varnothing$	
A'	"the complement of Set A" The complement of a set contains everything NOT included in the set.	The complement of A contains all elements of S that are not in A.	$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $A = \{1, 2, 3, 4\}$ $A' = \{5, 6, 7, 8, 9, 10\}$	

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PRACTICE

1. Draw a line from each number to the *smallest* subset that contains it.

REMEMBER: Rational numbers have a repeating decimal pattern, or they terminate. Any decimal pattern that does NOT repeat or terminate is an irrational number.



3

$\frac{1}{2}$

0.0125

$-2\frac{3}{17}$

$0.1011011101111...$

-5

0

$0.\overline{142857}$

$3.1415926535897...$

Natural numbers

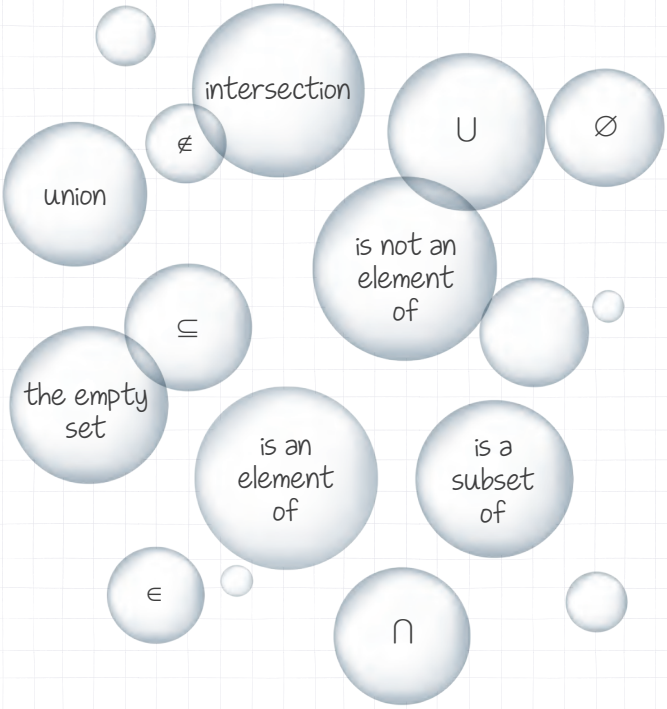
Whole numbers

Integers

Rational numbers

Irrational numbers

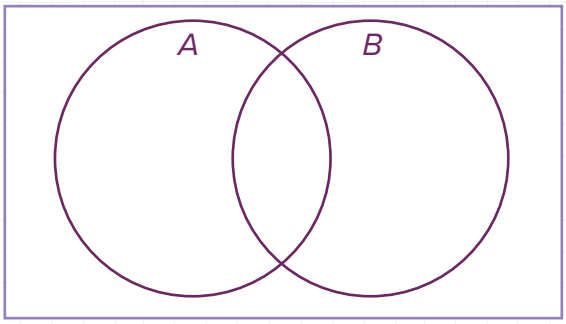
2. Color the bubbles with a symbol and its meaning the same color.



3. Shade the parts of each Venn diagram that correspond to each set. Then write the union or intersection on the line.

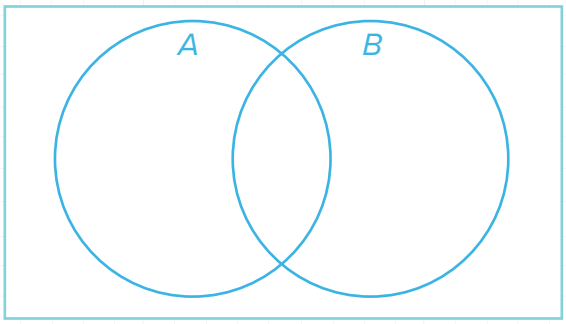
✦ Hint: Complements may be used with unions and intersections.

a. The set of things that are in A and also B



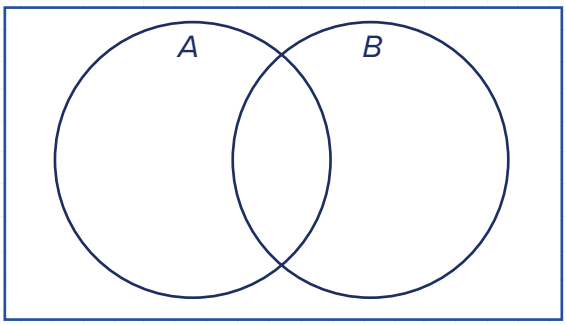
_____

b. The set of things that are in A but not B



_____

c. The set of things that are not in A and also not in B



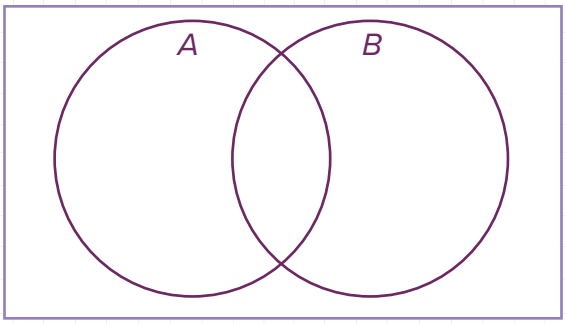
_____

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4. Complete the Venn diagram for the following sets.

$A = \{-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$

$B = \{\text{all prime numbers less than } 40\}$



Using set notation, determine the elements in the unions and intersections below. Remember to use curly braces around a set.

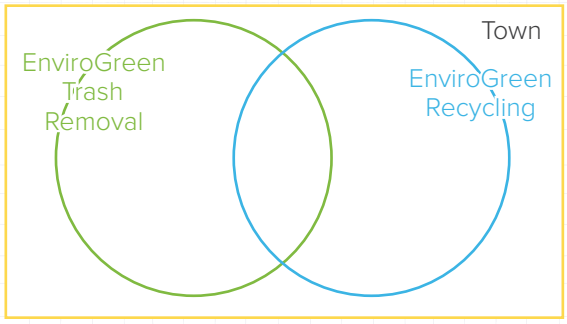
a. $A \cap B$ _____

b. $A \cup B$ _____

5. Complete the Venn diagram to model the situation below. Then answer the question.

In a town of 8,000 people, 5,400 use EnviroGreen for trash removal, 4,500 use EnviroGreen for recycling, and 3,900 use EnviroGreen for both.

✦ Hint: To find the number of people who use EnviroGreen for trash removal only, find 5400 – 3900. To find the number of people who use EnviroGreen for recycling only, subtract 4500 – 3900.



How many people do not use EnviroGreen's services? _____

✦ Hint: Add the three numbers in the Venn diagram and subtract the sum from 8,000 people.

REVIEW

1. Solve for the variable of each equation. L27

a. $2.4d = 12$ _____

b. $11.9 + f = -49.1$ _____

2. Write an equation to model each statement. L26

a. Fifteen less than one-eighth of n is equal to four more than the opposite of p.

_____

b. The product of q and r is nine times s.

_____

3. Evaluate each expression when $g = -3$ and $h = 27$. L24

a. $g - \sqrt[3]{h}$ _____

b. $6g + h$ _____

4. Rewrite each number in scientific notation. L18

a. 0.0002009 _____

b. 6,530,000,000 _____

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5. A garden store is having a 30%-off sale. Saving 30% is equivalent to paying 70% of the original price. Mentally find the sale price for each item by multiplying the original price by seven and then moving the decimal point one place to the left.



a. A shrub with an original price of $30

_____



b. A gardening book with an original price of $12

_____



c. A large basket of flowers with an original price of $50

_____

6. Fill in each blank with a number that makes the statement true. L16

a. $(-25+31)+8=____+(31+8)$

b. $23 \bullet ____ = 47 \bullet 23$

7. A number is divisible by 4 if the number formed by its last two digits is divisible by 4. Circle the numbers that are divisible by 4.

222	554
1,124	3,780
936	826







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PRACTICE

Solve the equations. Color the picture according to the solution value. A hint is given for a suggested first step for each problem, but the problems can be solved in other ways.

Blue

1. $-3x + 4 = 20 - x$ ✦ Hint: Add x to both sides.

Solution: _____

Red

5. $3r - 1 = r + 3$ ✦ Hint: Subtract r from both sides.

Solution: _____

Green

2. $3 - 2a + 5 = -12 + 2a$ ✦ Hint: Add $2a$ to both sides.

Solution: _____

Blue

6. $4y + 5 = 7 + y - 5$ ✦ Hint: Combine 7 and –5 on the right side.

Solution: _____

Yellow

3. $7 - 2b = 1 + b + 5 - 2b$ ✦ Hint: Combine b and $-2b$ on the right side.

Solution: _____

Green

7. $2 - 3z = z + 2$ ✦ Hint: Add $3z$ to both sides.

Solution: _____

Orange

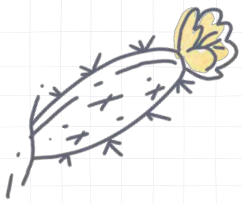
4. $12 + 14s = 72 - 6s$ ✦ Hint: Add $6s$ to both sides.

Solution: _____

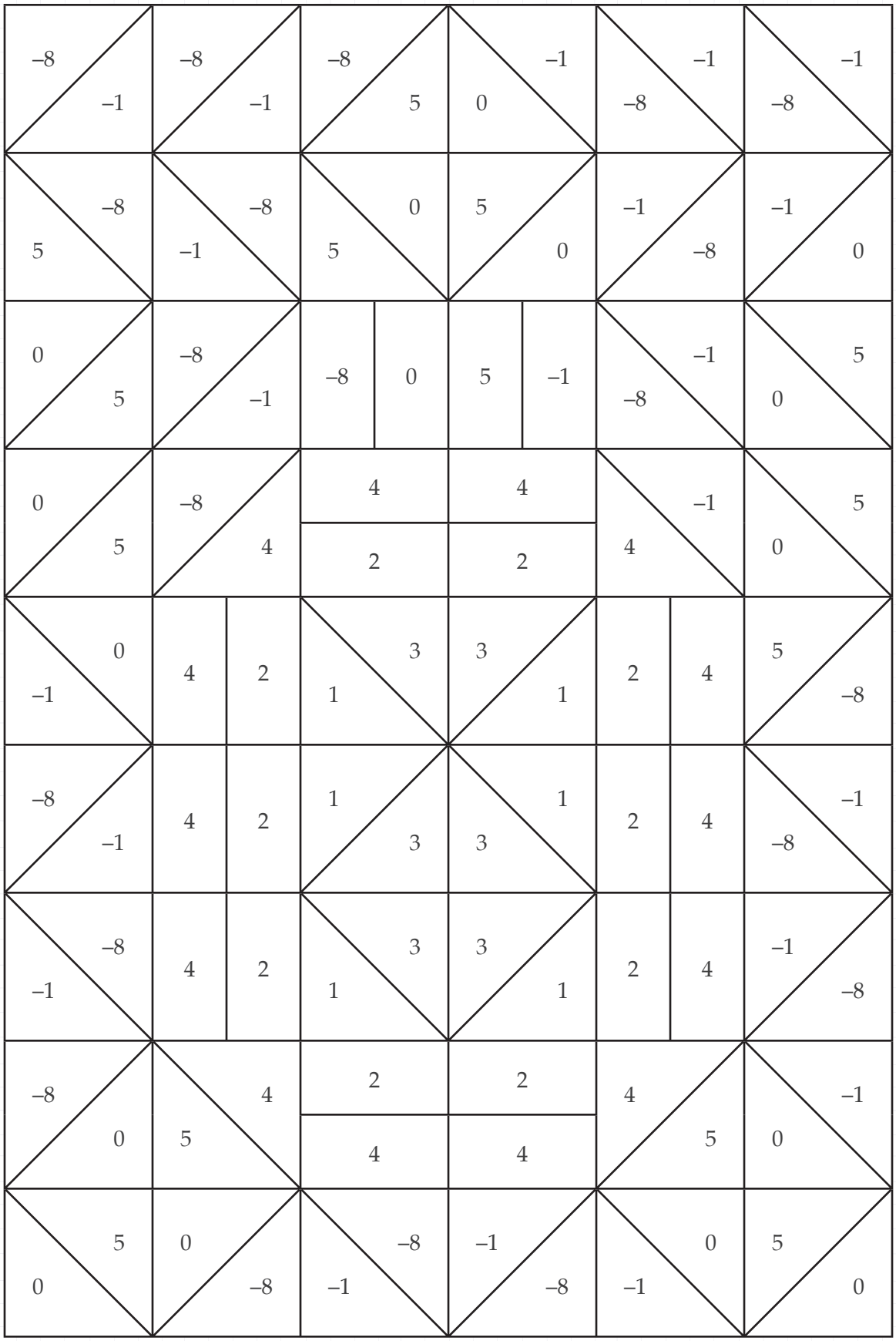
Brown

8. $p - 3 = 5 - p$ ✦ Hint: Add p to both sides.

Solution: _____



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–8 –1
–8 –1
–8 5
0 –1
–8 –1
–8 –1
5 –8
–1 –8
5 0
0 5
–8 –1
–1 0
0 5
–8 –1
–8 0
5 –1
–8 –1
0 5
0 5
–8 4
4 4
2 2
4 –1
0 5
–1 0
4 2
1 3
3 1
2 4
5 –8
–8 –1
4 2
1 3
3 1
2 4
–1 –8
–1 –8
4 2
1 3
3 1
2 4
–1 –8
–8 0
5 4
2 2
4 4
4 5
0 –1
0 5
0 –8
–1 –8
–1 0
5 0

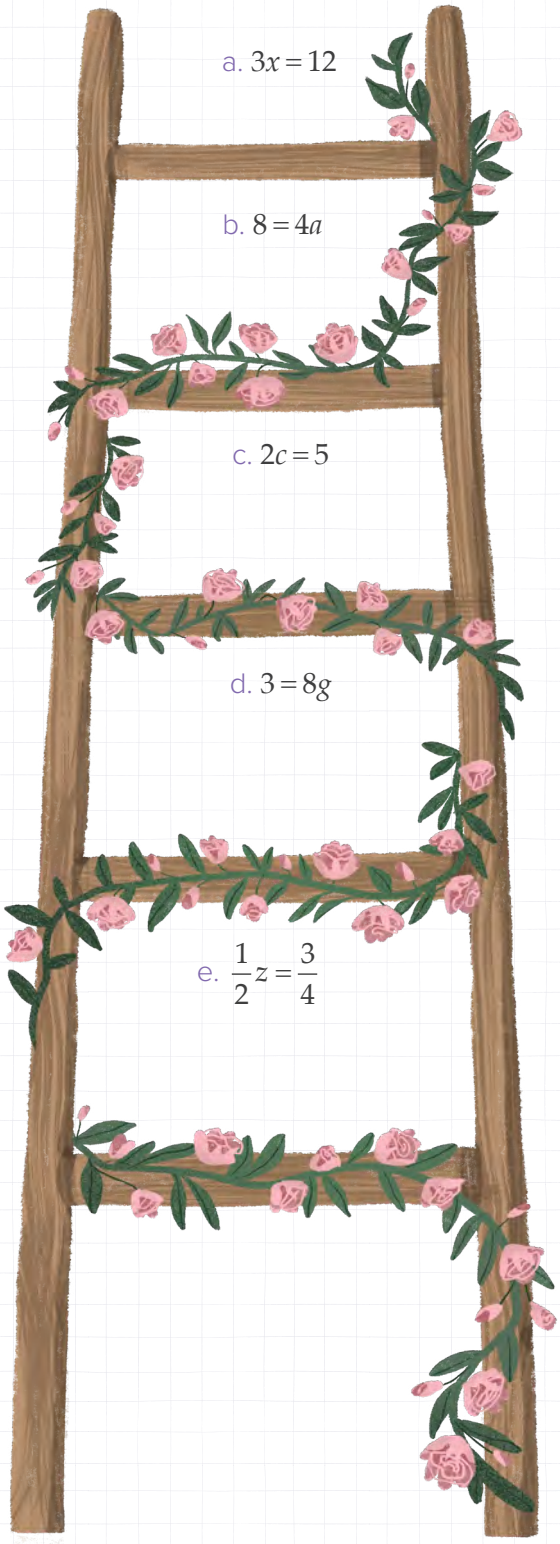
Solving Equations Review

There is no video for this lesson. The entire lesson is practice and applications for solving equations.

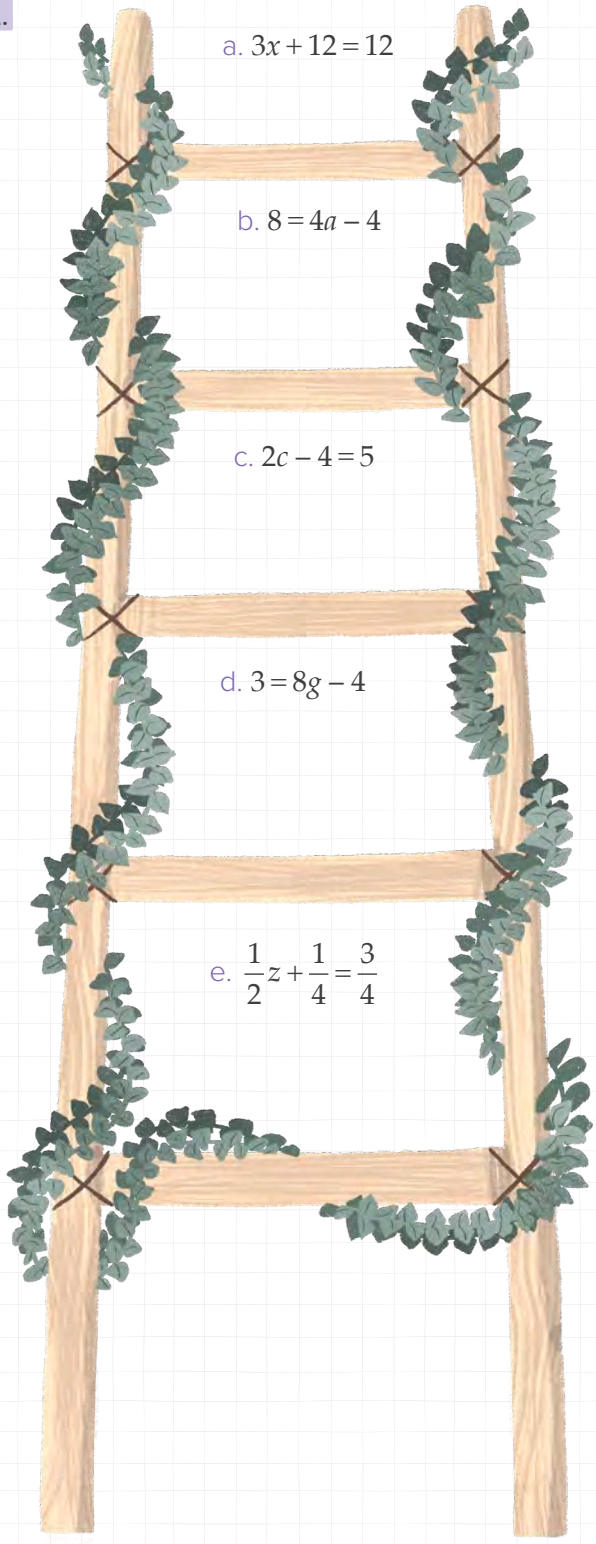
LADDER TO SUCCESS

Solve the problems on each ladder.

1.



2.



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UNIT 2 | LESSON 39

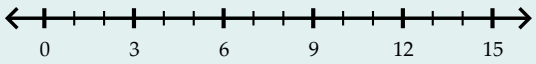
Solving and Graphing Multi-Step Inequalities

SUPPLIES: colored pencils and/or highlighters

WARM-UP

Solve and graph the inequality.

$$x - 10 > -3$$



LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Notes section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

App Video

VIDEO NOTES

When solving multi-step inequalities, perform inverse operations in the same order as when solving equations.

1. Add or subtract _____ to isolate the term with the variable.
2. Divide (or multiply by the reciprocal of) _____ to isolate the variable on one side.

Note: Like terms may need to be combined before completing these steps.

$$4 + 1.5r \leq 25$$

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LESSON OVERVIEW

As when solving equations, multiple steps may be required to solve an inequality. When solving multi-step inequalities, perform inverse operations in the same order as when solving equations. Note that like terms may need to be combined before completing these steps.

1. Add or subtract constants to isolate the term with the variable.
2. Divide (or multiply by the reciprocal of) coefficients to isolate the variable on one side.

KEY INFORMATION

- Any operation done to one side of the inequality must also be done to the other side of the inequality.
- When multiplying or dividing by a negative, switch the direction of the inequality sign.

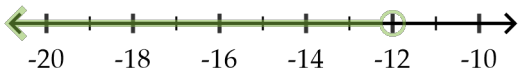
Example 1: $\frac{4}{3}x - 16 < -32$

$\frac{4}{3}x - 16 + 16 < -32 + 16$ Add 16 to both sides.

$\frac{4}{3}x < -16$

$\frac{3}{4} \cdot \frac{4}{3}x < -16 \cdot \frac{3}{4}$ Dividing by $\frac{4}{3}$ is the same as multiplying by the reciprocal. The reciprocal of $\frac{4}{3}$ is $\frac{3}{4}$.

$x < -12$ The graph of the inequality has an **open circle** at –12 because the inequality is less than, not less than *or equal to*, –12. x is any number less than –12.



Example 2: $-6p + 25 \geq 49$

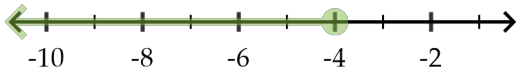
$-6p + 25 - 25 \geq 49 - 25$ Subtract 25 from both sides.

$-6p \geq 24$ Divide both sides by –6.

$\frac{-6p}{-6} \geq \frac{24}{-6}$

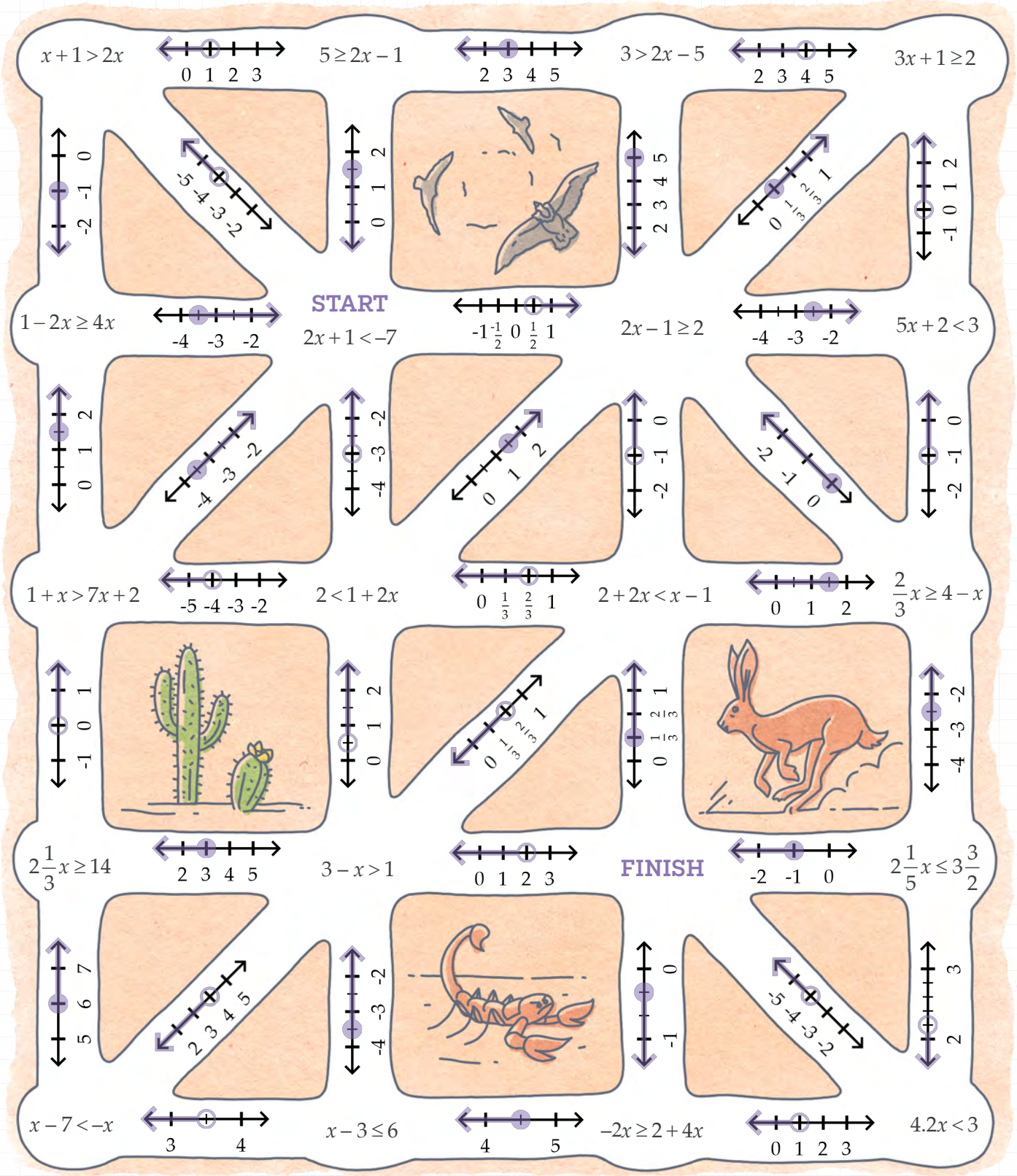
$p \leq -4$ The graph has a **closed circle** at –4 because p is less than *or equal to* –4. p can be any number less than –4, or p can equal –4.

The inequality sign switches directions when dividing by a negative.



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4. Begin at the START. Solve the inequality. Then find the number line that shows the solution and follow that path to the next inequality to solve. Continue following the number line solutions until you reach the FINISH.



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PRACTICE



1. If $\frac{1}{4}$ of the peaches are white and there are 51 white peaches, how many peaches are there altogether?

   ✦ Hint: 51 is $\frac{1}{4}$ of what number?

   _____

2. There are 30 lb of baby carrots. If baby carrots make up $\frac{3}{8}$ of the total pounds of carrots, how many pounds of carrots are there?

   ✦ Hint: 30 is $\frac{3}{8}$ of what number?

   _____

3. $\frac{5}{6}$ of the cabbage heads are napa cabbage. There are 35 heads of napa cabbage. How many total cabbage heads are there?

   ✦ Hint: 35 is $\frac{5}{6}$ of what number?

   _____

4. In a basket of onions, $\frac{2}{7}$ of the onions are red onions. If there are 56 onions in the basket, how many are red onions?

   ✦ Hint: What is $\frac{2}{7}$ of 56?

   _____

5. $\frac{5}{12}$ of the 84 apples are yellow apples. How many yellow apples are there?

   ✦ Hint: What is $\frac{5}{12}$ of 84?

   _____

6. There are 28 heads of lettuce. $\frac{3}{4}$ of the heads are iceberg. How many heads of iceberg lettuce are there?

   ✦ Hint: What is $\frac{3}{4}$ of 28?

   _____

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Salik's Squares

In Greenland the northern lights, or *aurora borealis*, are best viewed on clear winter nights. The colorful lights, which are caused by high-energy particles from the sun colliding with particles in the earth's upper atmosphere, appear to dance across the sky.

Suppose that you are visiting Greenland and are excitedly waiting to see the northern lights. A young Greenlander named Salik suggests that you pass the time by solving a puzzle he created.

Each row, column, and 3x3 square must contain the digits 1 through 9 exactly once. Fill in the missing numbers in the puzzle.

7		3		4	1	8		6
1			9			3		4
8	5	4	3		2	1	9	7
	4	8	1	2		9		5
	1		6		3		4	
9	3			5		6	7	
3		5	7				6	2
2	6	9		8	5			3
	7			3	6	5	8	9

Hint: Start with a row, column, or 3x3 square that is mostly filled in. Go through each number (1 through 9) to see what is missing and where it could go. Be sure that writing a number will not cause that same number to appear in a row, column, or 3x3 square more than once.

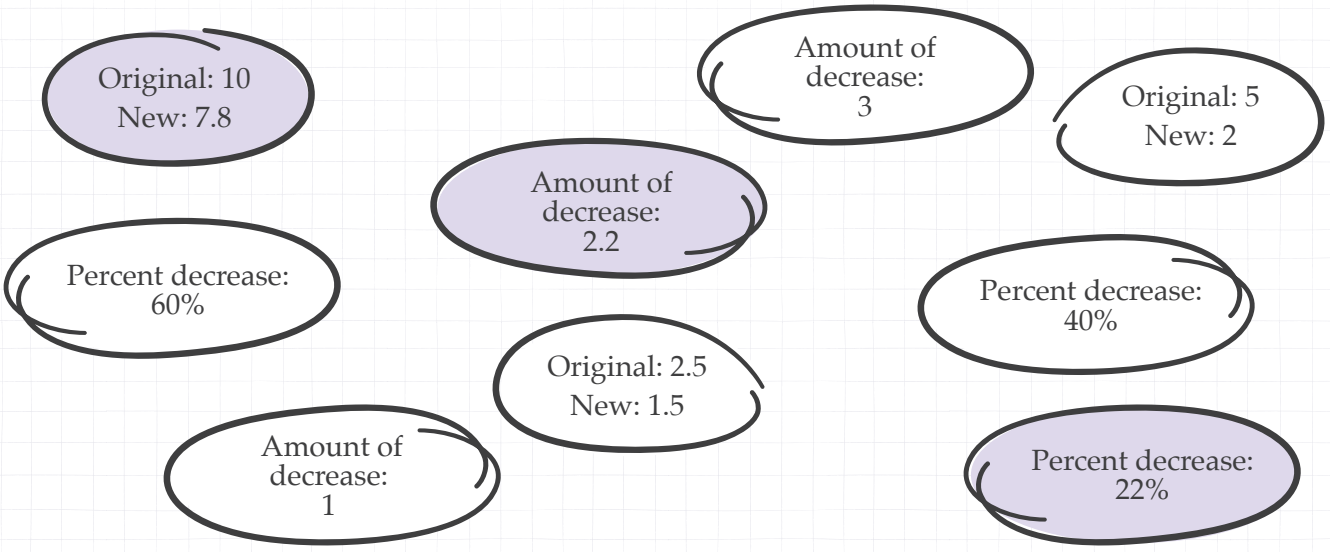
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PRACTICE

A calculator may be used for this entire practice section.

Finding the Percent Decrease

1. Color in the matching bubbles to show which new and original amounts correspond to which amount of decrease and percent decrease. An example is given.

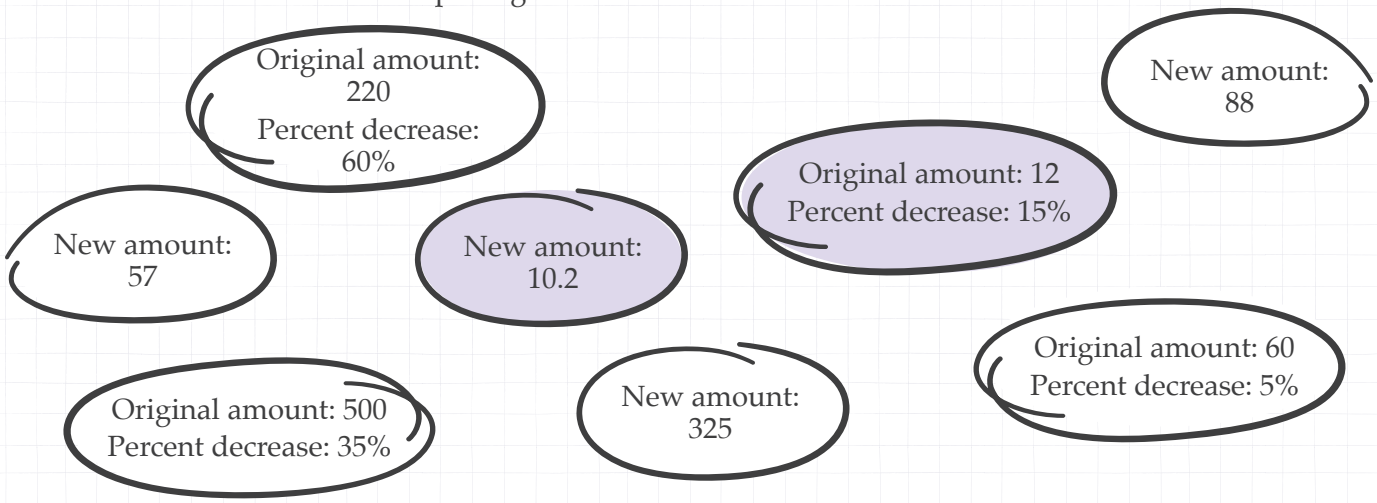


2. A computer costs $1,200 new but is only worth $570 two years later. Find the percent decrease in the value of the computer.

_____

Finding the New Amount

3. Color in the matching bubbles to show which original amount and percent decrease correspond to which new amount. An example is given.

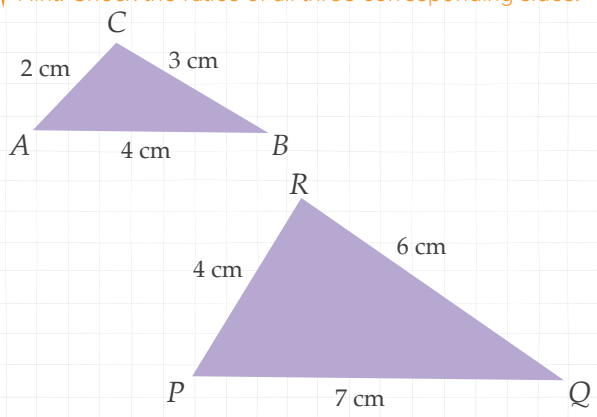


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3. Use ratios to determine if the triangles are similar. Write *yes* or *no* on the line.

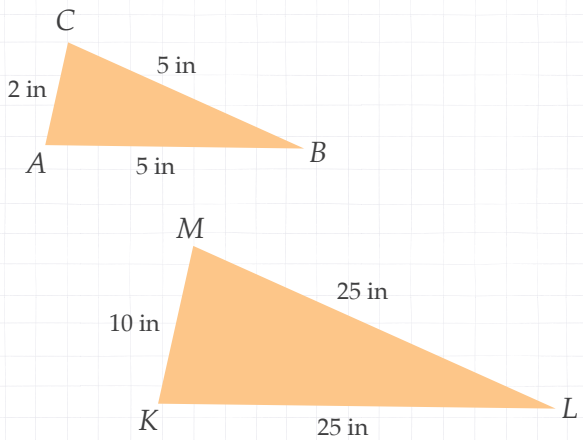
✦ Hint: Check the ratios of all three corresponding sides.

a.



_____

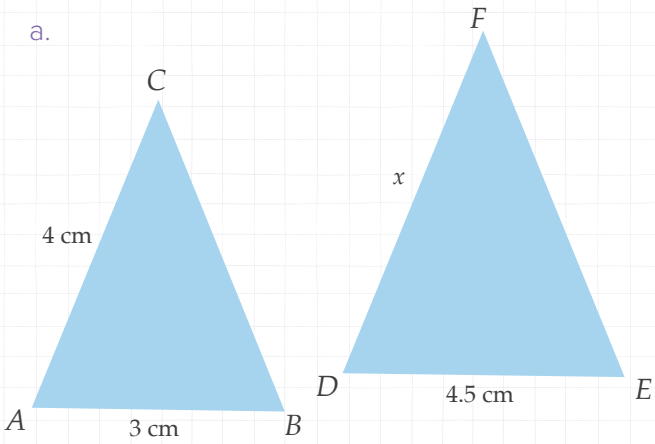
b.



_____

4. Find the missing side lengths for the proportional triangles.

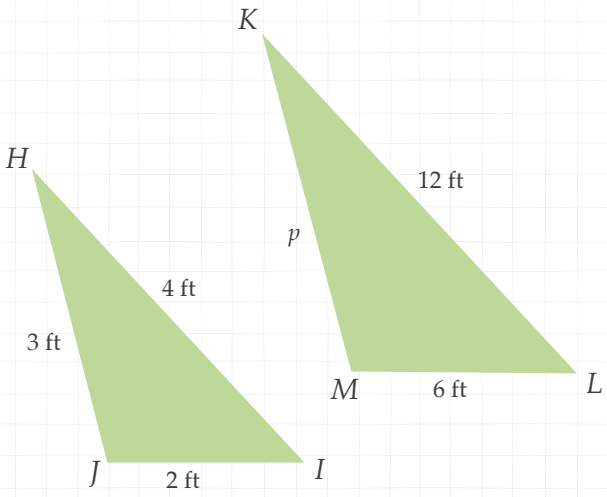
a.



$\triangle ABC \sim \triangle DEF$

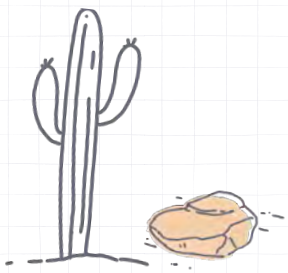
$x =$ _____

b.



$\triangle HIJ \sim \triangle KLM$

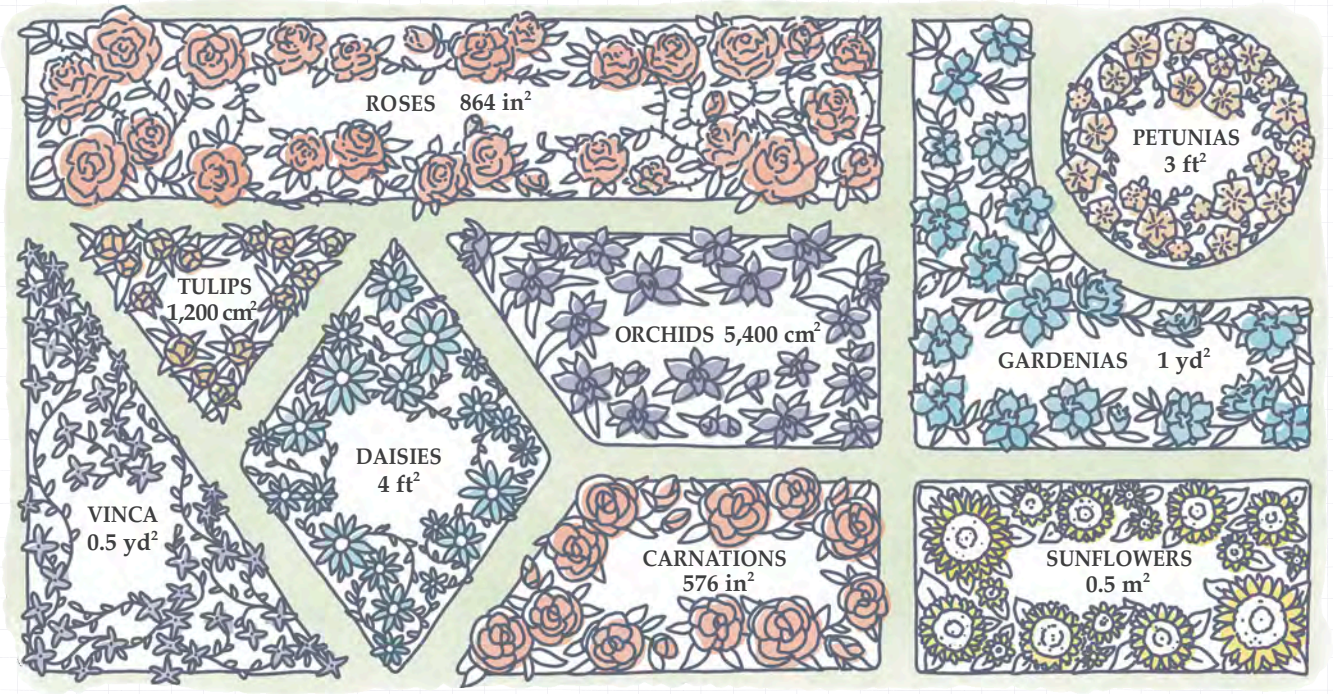
$p =$ _____



PRACTICE

A calculator may be used for this entire practice section.

A family collaborated on a blueprint for a garden, but they all used different units! Use unit multipliers and convert between units to answer the questions below.



1. a. Convert the area of the rose bed into square feet.

_____

b. How much more space do the roses have than the daisies?

_____

2. a. Convert the area of the vinca bed into square feet.

_____

b. Do the petunias or the vinca have more space? How much more space?

_____

_____

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Mixed Review

This lesson is a mixed review. There is no video, practice, or review section.
You may use a calculator for this entire review activity.

SHARPEN YOUR SKILLS!

Suppose your neighborhood is having a potluck, and your family volunteered to bring a pan of enchiladas to the party. It is your responsibility to buy all the ingredients for the enchiladas and stay within budget.

Your budget is $35, including sales tax. Tax is 3% of the total cost. Note: Some US states do not charge sales tax on groceries. For this review activity, suppose groceries are taxed at 3%.

The lists below show the ingredients you must purchase, as well as optional ingredients.

Required ingredients

- 12 tortillas
- 2 lb of meat
- $\frac{3}{4}$ lb of cheese
- 3 cans of beans
- 3 jars of sauce

Optional ingredients

- Additional tortillas (Up to 16 tortillas can fit in one pan.)
- 1–2 lb of extra meat
- $\frac{1}{4}$ lb of extra cheese
- Additional cans of beans
- Up to 3 cans of chiles
- 1 extra jar of sauce
- Onion
- Sour cream
- Olives

Instructions

- Use the illustration on the following page to choose your ingredients. The illustration page can be removed from the book if desired. Record your choice for each ingredient variety in the second table. Record any optional ingredients you choose to include in the rows labeled "extra ingredient."
- Write the given unit cost of one item in the Unit Cost column. Under Quantity, write how many/much of each item you plan to buy.
- Multiply the Unit Cost by the Quantity to get the Ingredient Cost.
  Unit Cost • Quantity = Ingredient Cost
- Add the Ingredient Costs to get the Subtotal.
- Find 3% of the Subtotal to get the Tax Amount. Multiply the Subtotal by 0.03.
  Subtotal • 0.03 = Tax Amount
- When finding the Tax Amount, round to the nearest hundredth. For example, 13.59 • 0.03 = 0.4077. The tax rounded to the nearest hundredth is $0.41.
- Add the Subtotal and the Tax Amount to get the TOTAL. Make sure your TOTAL is less than your budget of $35. If your total is over budget, eliminate or change ingredients.

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Unit 2 Review

Complete this Unit Review to prepare for the Unit Assessment. There is no video, lesson, or practice. Because Unit Reviews include practice for an entire unit, they may take longer than regular lessons, and students may decide to take two days to finish.

Chinese New Year

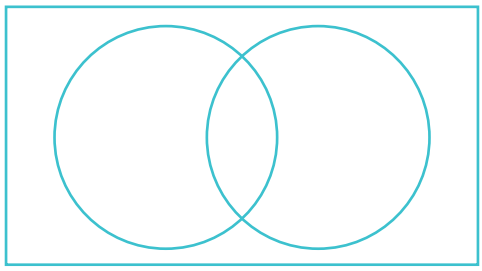
Chinese New Year is based on a lunisolar calendar and typically begins at the new moon, which falls between January 21 and February 20. After completing each problem, copy the word that is underlined onto the line with its problem number at the end of the review.

Venn Diagrams and Set Notation
Lesson 31

1. For a village gathering, everyone votes to have spring rolls with either pork or shrimp. There are 63 people who want shrimp, 45 people who want pork, and 34 people who want both shrimp and pork.

   a. Create a Venn diagram to model this situation.

   ✦ Hint: If 63 people want shrimp, this includes people who want both.

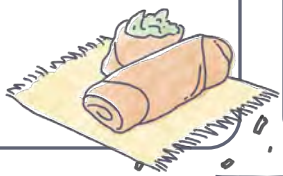


   b. Use the Venn diagram to determine how many villagers there are.

   _____ villagers

   c. If A is the set of people who want shrimp, and B is the set of people who want pork, use set notation to write the set of people who would like both pork and shrimp.

   _____



Square and Cube Roots
Lessons 32, 34



2. Lotus root is commonly eaten during the new year festival to symbolize abundance. Answer these questions about mathematical roots!

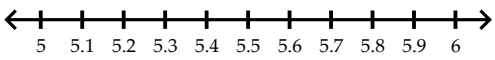
   a. Determine which two perfect squares are closest to 33. Circle the one that is closer to 33.

   _____ and _____

   b. Calculate $\sqrt{33}$ rounded to the nearest hundredth.

   $\sqrt{33} \approx$ ____

   c. Plot the approximate value of $\sqrt{33}$ on the number line.



3. Find the following roots.

   a. $\sqrt{81}$ _____

   b. $\sqrt[3]{64}$ _____

   c. $\sqrt[3]{-27}$ _____

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Other Fun Facts About the Chinese New Year and Its Symbols

- The Chinese New Year celebration is also called the _____ festival.
  1

- _____ flowers grow in the mud.
  2
  They are day-blooming flowers. This means they curl their petals back into the mud at night.



- Besides fish and _____ envelopes,
  14
  other _____ symbols associated with
  4
  the Chinese New Year are three coins tied with a ribbon, bamboo plants, and fruit blossoms.



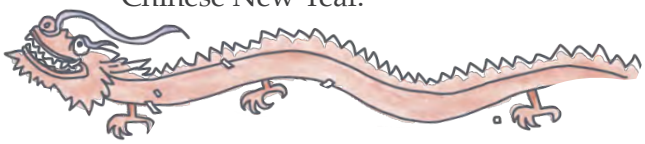
- A Chinese saying to wish someone well on an exam is to say, "A _____ leaping
  5
  over the dragon gate!"

- Chinese families may set off _____
  7
  when opening a door for the first time in a day. It symbolizes good fortune for the whole year.



- China is credited with the invention of _____.
  9
  A legend tells of a Chinese scientist who invented _____
  9
  by accident while mixing charcoal, sulfur, and saltpeter. After being pressed into a bamboo tube, the mixture exploded!

- The Chinese word for _____
  11
  sounds like "luck," and the Chinese word for _____
  12
  sounds like "wealth." Both of these fruits are given as gifts and used as decorations during the Chinese New Year.



- More Chinese children tend to be born during the years of the _____
  16
  than during other years.

- Another popular tradition is to put a riddle inside a _____
  17
  for others to solve.

- The filling for _____
  20
  usually consists of ground nuts or sesame seeds, sugar, and lard.

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Unit 2 Assessment

- This assessment covers concepts taught in Unit 2. Problems are designed to assess multiple skills. Read the instructions carefully and do not rush through the problems.
- You may use the Reference Chart at the back of the book for the assessment. Calculators should be used only when noted.
- Lesson numbers are given by each problem so you can review lessons for any answers that are incorrect.

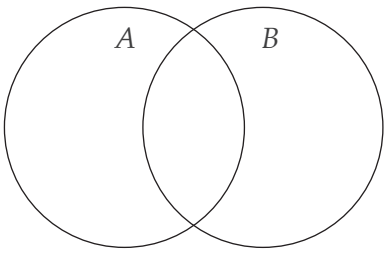
1. Match the number with the smallest set the number belongs in. L31

3	rational $(\mathbb{Q})$
0	whole numbers (W)
-11	irrational (I)
$\frac{1}{2}$	natural numbers $(\mathbb{N})$
$\sqrt{2}$	integers $(\mathbb{Z})$

2. Fill in the Venn diagram and find $A \cap B$. L31

$A: \{1,2,3,4,5,6,7,8,9,10\}$

$B: \{2,4,6,8,10,12,14,16,18,20\}$



_____

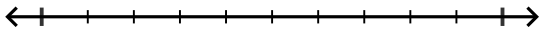
3. Write the two integers each square root falls between. L32

a. $\sqrt{56}$ _____ and _____

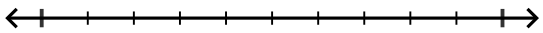
b. $\sqrt{11}$ _____ and _____

4. Calculate the value of each square root in Problem 3 (rounded to the nearest hundredth). Then plot the rounded value on a number line. L32

a. $\sqrt{56} \approx$ _____



b. $\sqrt{11} \approx$ _____



5. Solve the following equations. L33, L35

a. $5x - 10 = 45$ _____

b. $\frac{1}{2}z - \frac{5}{6} = \frac{1}{6}$ _____

c. $a + 5 = 6a$ _____

d. $7 - 4c = 8c + 17$ _____

Enrichment: Graph Theory

This is an enrichment lesson. Mastery is not expected at this level.
There is no video, practice, or review.

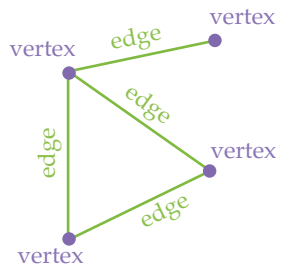
SUPPLIES: colored pencils

Modeling with Graph Theory

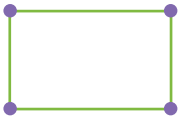
What do you think of when you hear the word "graph"? Maybe you think of a number line or coordinate plane? Maybe you think of a bar, line, or circle graph? In graph theory, we study graphs made out of *vertices* (singular: one *vertex*) and *edges*.

Each of the following is a graph. In graph theory, the *vertices* are points (or dots), and the *edges* are lines connecting those vertices.

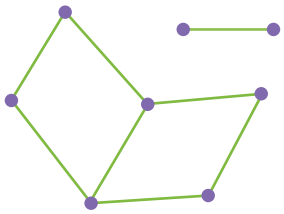
Example 1:



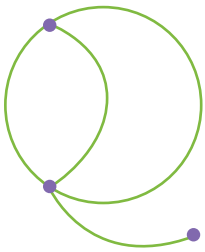
Example 2:



Example 3:

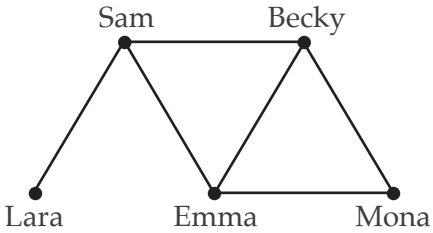


Example 4:



Graphs can be used to model many things, like relationships and transportation networks. In these models, the vertices usually represent some kind of object, and the edges represent relationships between those objects.

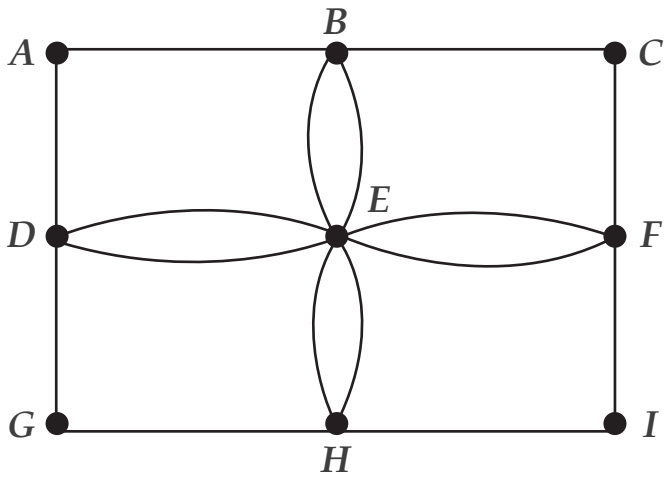
Example 5:



Becky, Emma, and Mona are in the same ballet class. Becky is also in band with Emma and Sam. Sam and Lara take French lessons together. On the left is a graph that represents this situation; edges show the relationship "have a class together."

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Label the edges "1," "2," etc., to help the mail carrier find a mail delivery route that begins and ends at his truck (*A*) and lets him deliver mail to all the houses without having to walk any sidewalk more than once.



Graph Theory and Maps

Another application of graph theory is maps! Think of the maps that you have seen. Neighboring countries or states are usually colored with different colors to make the borders easy to see. A natural question when creating a map is how many colors are needed to color a map so that neighboring regions are always different colors. We consider regions to be neighbors if they actually share part of a border (not just meeting at corners).

Here are some examples of maps where neighboring regions are colored differently.

Example 8:



Example 9:



Example 10:



Notice that these examples use many different colors to color the maps.

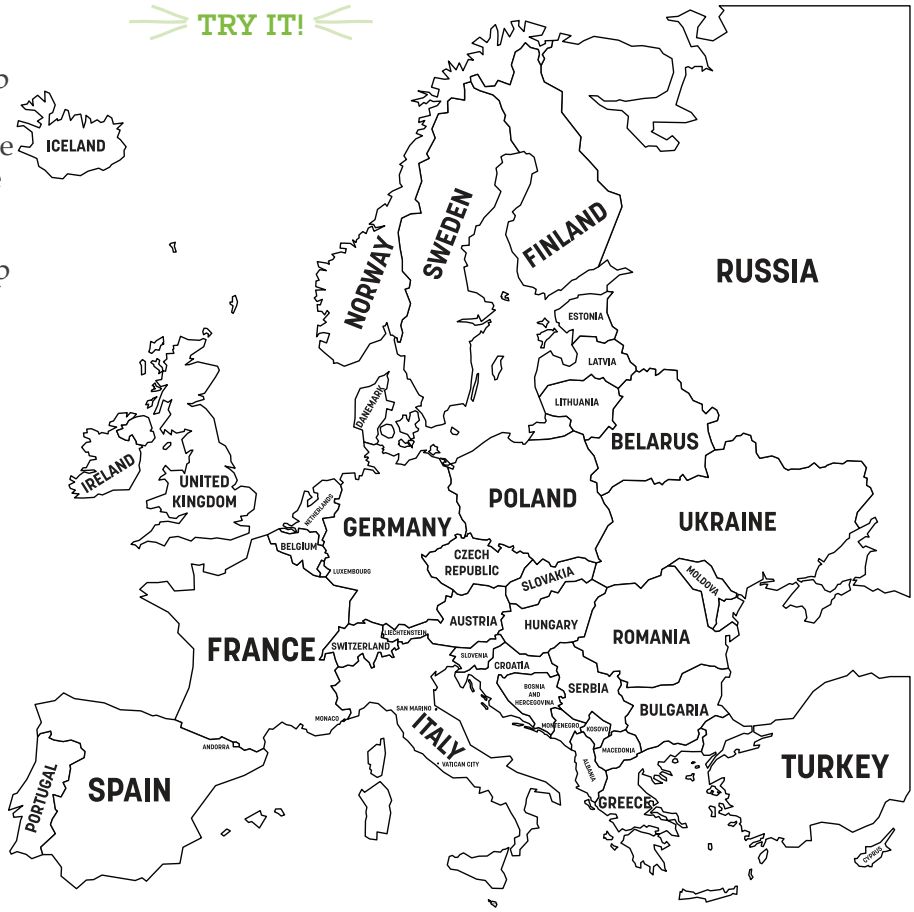
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Note: You may want to make a copy of this page before completing the next two activities so you can try it multiple times!

TRY IT!

4. Can you color the same map as in Example 10, but with FEWER colors, and still have all neighboring countries be different colors?

   Remember, the game of map coloring is to use as FEW colors as possible while still having neighbors be different colors.



5. Try coloring the maps below with the FEWEST colors so that neighboring states are always different colors. Write how many colors you need in each case.

a. _____



b. _____



Note: For this graph, only color states that are labeled. Do not color the upper region of Michigan that is not labeled.

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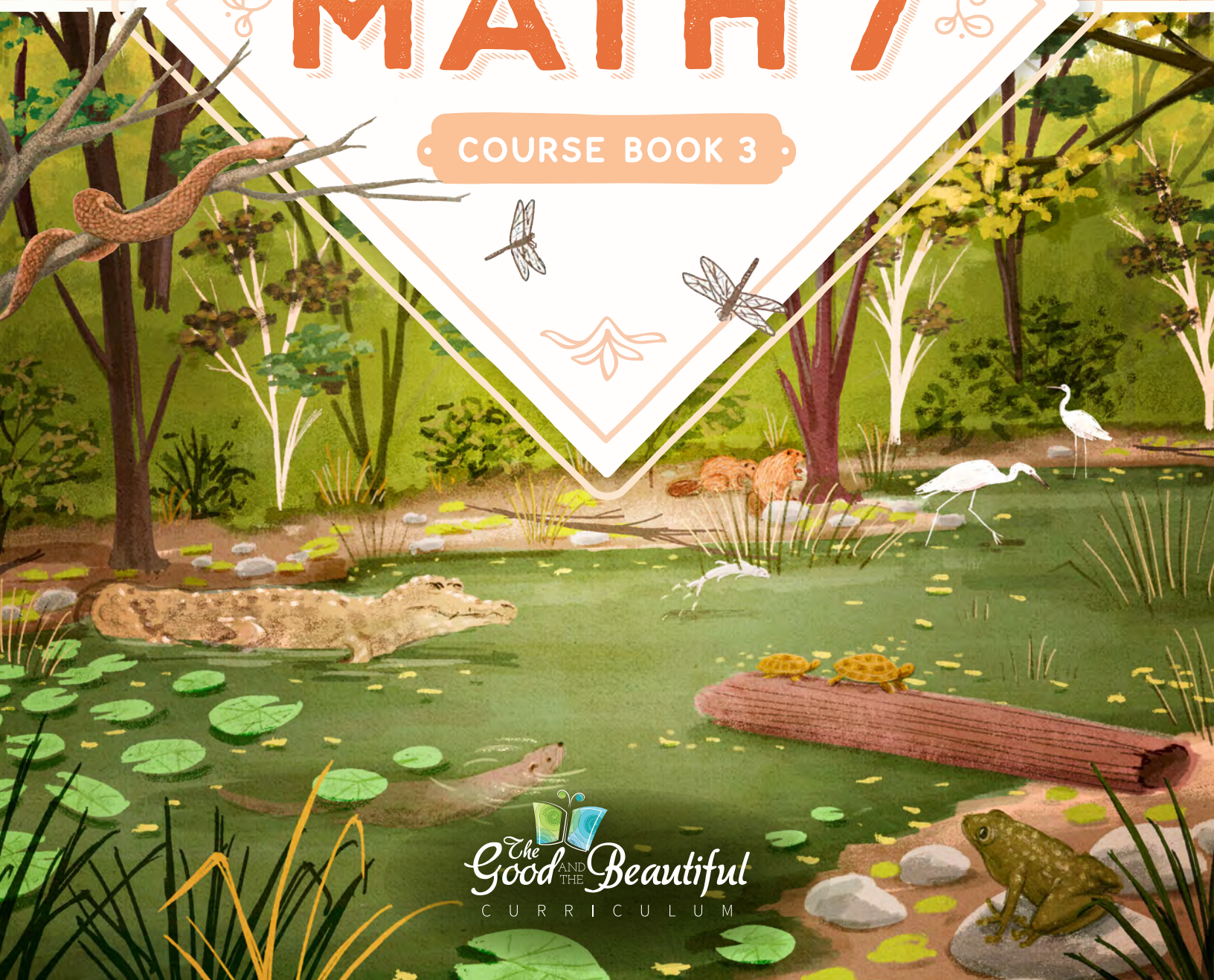
LESSONS
61–90

UNIT
3

Simply Good and Beautiful

MATH 7

COURSE BOOK 3



The Good and the Beautiful
CURRICULUM

COURSE BOOK 3

TABLE OF CONTENTS

UNIT 3 OVERVIEW 1

Lesson 61: Scale Drawings 2
Lesson 62: Direct Proportions 8
Lesson 63: Inverse Proportions 13
Lesson 64: Graphs of Direct Proportions 19
Lesson 65: Graphing Using a T-Chart 25
Lesson 66: Slope of a Line 31
Lesson 67: Slope-Intercept Form 39
Lesson 68: Graphing Linear Equations 45
Lesson 69: Functions 52
Lesson 70: Graphing Functions 57
Lesson 71: Triangles 63
Lesson 72: Transformations 69
Lesson 73: Constructing Angles 76
Lesson 74: Constructing Triangles 83
Lesson 75: Logic Lesson 3 89
Lesson 76: Polygon Diagonals and Angles 92
Lesson 77: Finding Polygon Angle Measures 98
Lesson 78: Angle Relationships 104
Lesson 79: Parallel Lines and Transversals 111
Lesson 80: Missing Angles in a Circle 117
Lesson 81: Pythagorean Theorem 123
Lesson 82: Perimeter of Polygons 129
Lesson 83: Area of Polygons 136
Lesson 84: Area and Circumference of Circles 143
Lesson 85: Composite Figures 149
Lesson 86: Inscribed Shapes 156
Lesson 87: Mixed Review 162
Lesson 88: Unit 3 Review 166
Lesson 89: Unit 3 Assessment 172
Lesson 90: Enrichment: Circumference and Diameter 178

Reference Chart 185

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UNIT 3 OVERVIEW

LESSONS 61–90

CONCEPTS COVERED

- Alternate exterior angles
- Alternate interior angles
- Angles in a circle
- Area around inscribed shapes
- Area of circles
- Area of composite figures
- Area of triangles, parallelograms, and trapezoids
- Calculating slope from a graph
- Circumference of circles
- Classifying triangles by angles
- Classifying triangles by sides
- Complementary angles
- Constructing triangles given three angles
- Constructing triangles given three sides
- Corresponding angles
- Degrees of rotational symmetry
- Direct proportions
- Drawing angles with a protractor
- Expressions within angle pairs
- Finding angle bisectors with a compass
- Finding missing side lengths in right triangles
- Finding missing side lengths of polygons given area
- Finding perpendicular bisectors with a compass
- Finding proportionality constants on graphs
- Functions
- Graphing functions from tables
- Graphing linear equations using slope-intercept form
- Graphing linear equations using T-charts
- Graphs of direct proportions
- Identifying and using scale factors
- Identifying equations of functions
- Identifying function rules
- Inscribed shapes
- Interior angle sums
- Inverse proportions
- Isosceles trapezoid angle properties
- Lines of symmetry
- Measuring angles with a protractor
- Missing angles in a quadrilateral
- Missing interior angles of triangles
- Missing sides in composite figures
- Nonlinear functions
- Parallel lines cut by a transversal
- Parallelogram angle properties
- Perimeter and area of semicircles
- Perimeter of composite figures
- Perimeter of polygons
- Polygon diagonals
- Polygons with expressions as side lengths
- Properties of triangle angles
- Properties of triangle sides
- Proportionality constants
- Pythagorean Theorem
- Pythagorean triples
- Relationships of angles in a circle
- Rotational symmetry
- Scales and scale drawings
- Slope of a line
- Supplementary angles
- Transformations on the coordinate plane
- Transformations (rotations, reflections, translations)
- Using a compass
- Vertical angles
- Vertical line test
- Writing equations of graphs in slope-intercept form
- *x*- and *y*-intercepts

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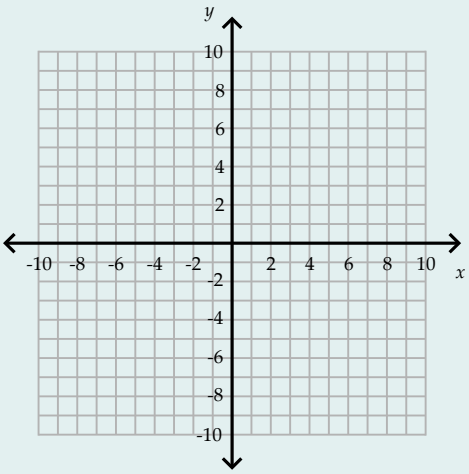
Slope of a Line

WARM-UP

Plot the ordered pairs on the coordinate plane.

a. $(2,5)$ b. $(-3,-1)$

c. $(-8,9)$ d. $(6,-4)$

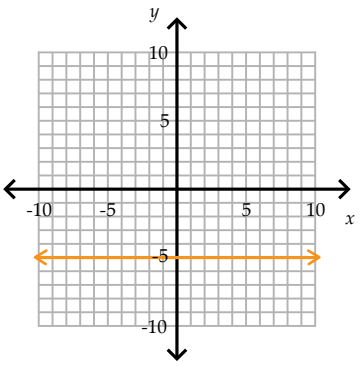


LESSON

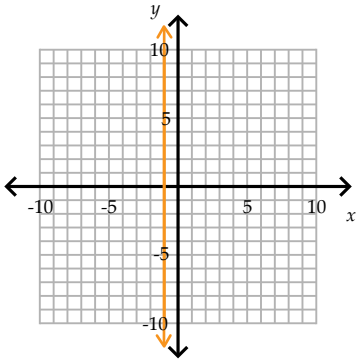
Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Notes section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

App Video

VIDEO NOTES

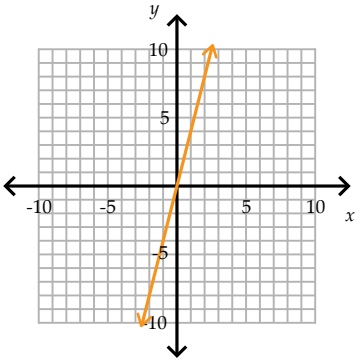


Positive slope

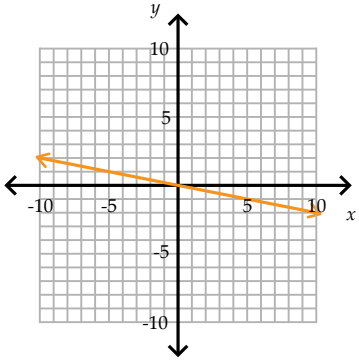


Negative slope

Undefined slope



Slope of zero

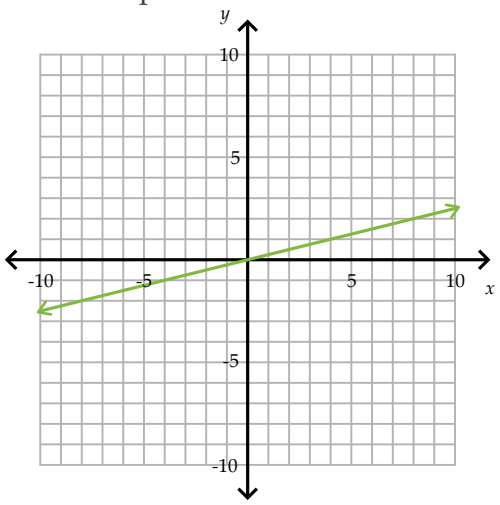


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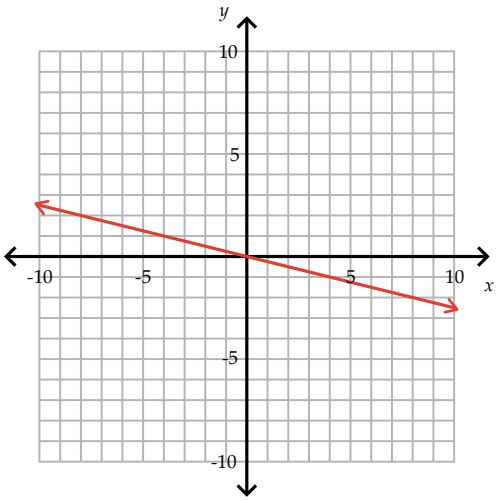
LESSON OVERVIEW

Types of Slope

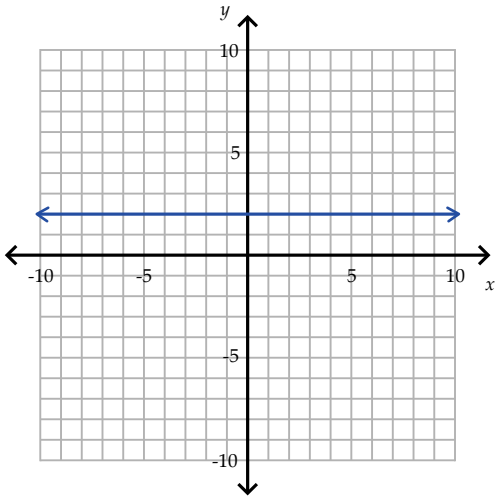
The ***slope*** of a line is the numerical change in y-values divided by the numerical change in x-values. Slope is also referred to as *rise* over *run*, where the *rise* is the *vertical* change in y-values and the *run* is the *horizontal* change in x-values. The slope of a line can be thought of as the steepness of the line.



This graph has a positive slope. A positive slope is seen when the slant of the line goes upward from left to right. As the x-values increase, the y-values also increase.



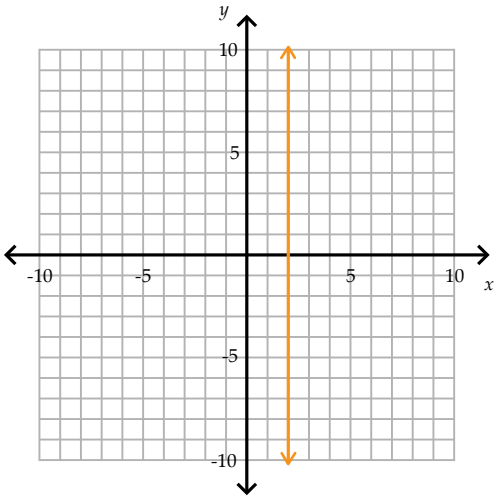
This graph has a negative slope. A negative slope is seen when the slant of the line goes downward from left to right. As the x-values increase, the y-values decrease.



This graph has a slope of zero. Any horizontal line (flat across, no slant) has a slope of zero. As the x-values increase, the value of y does not change.

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This graph has no slope, which also means the slope is undefined. A line with an undefined slope is a vertical line (up and down). The value of x stays the same for all y-values.



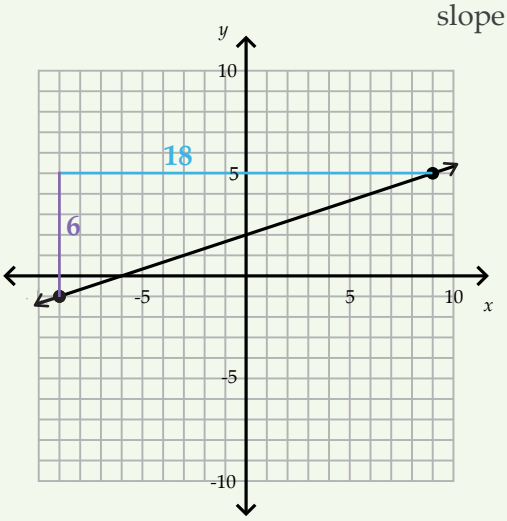
Calculating Slope

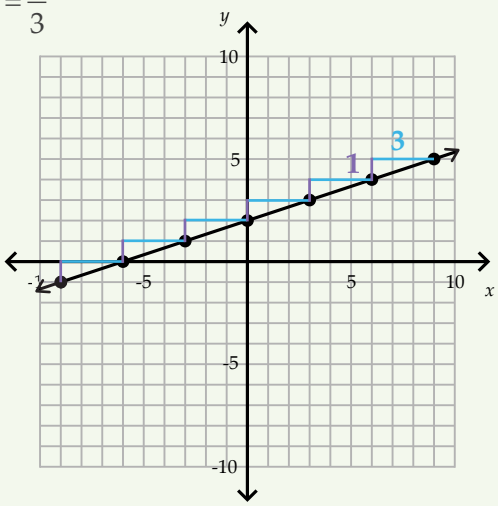
Slope can be calculated on a graph by counting the rise (vertical distance) and run (horizontal distance) between *any* two points on the line. When counting rise and run, start with a point on the left and count as you move toward a point on the right. Write the slope as a fraction of rise over run and simplify.

Positive Slope

Look at the graph on the left below. Start at the ordered pair $(-9,-1)$. Count units going *up* (rise) until the height of the next point is reached. The rise is 6. Then count units going *right* (run) until the point is reached. The run is 18. The slope is the fraction formed by the rise divided by the run.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{6}{18} = \frac{1}{3}$$





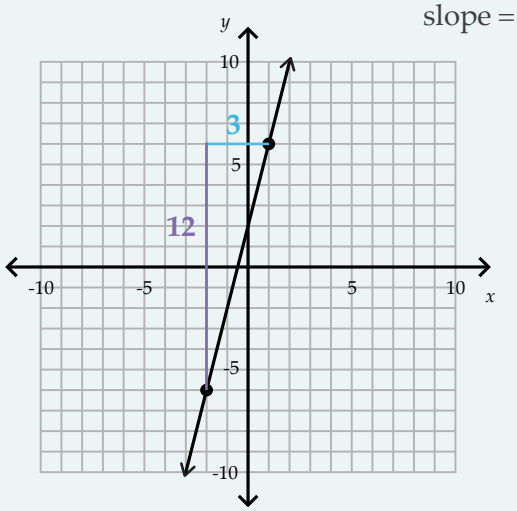
The slope of $\frac{1}{3}$ can be seen in the graph on the right. For each rise of 1, there is a run of 3. The slope between any two points will simplify to $\frac{1}{3}$.

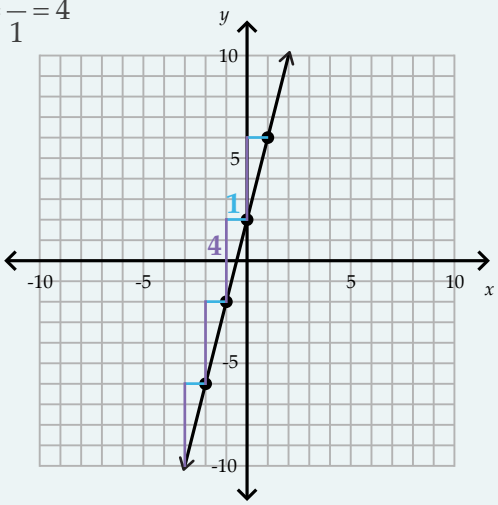
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Integer Slope (Positive or Negative)

Look at the graph on the left below. Going up from the point $(-2,-6)$, the rise is 12. Going to the right to the point $(1,6)$, the run is 3. In this example the slope reduces to an integer.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{12}{3} = \frac{4}{1} = 4$$



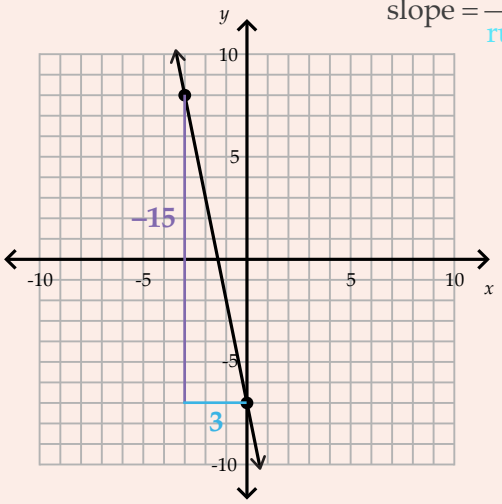


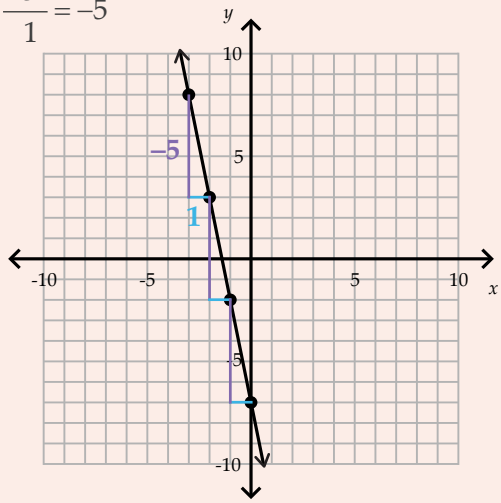
The slope of 4 can be seen in the graph on the right. For each rise of 4, there is a run of 1. The slope between any two points will simplify to 4.

Negative Slope

The rise and run of negative slopes can be counted down and to the right. Look at the graph on the left below. Start at $(-3,8)$ and count *down* (rise) to the next point. The rise is negative because we are counting units moving down the graph. The rise is –15. Count to the *right* (run) until the next point is reached. The run is 3. The slope (rise over run) is negative because a negative number divided by a positive number is a negative number.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-15}{3} = \frac{-5}{1} = -5$$





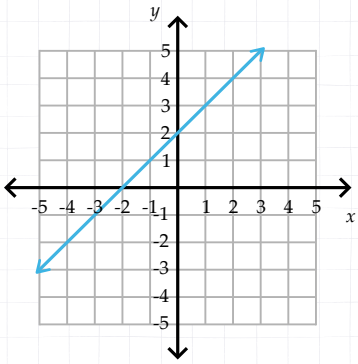
The slope of –5 can be seen in the graph on the right. For each rise of –5, there is a run of 1. The slope between any two points will simplify to –5.

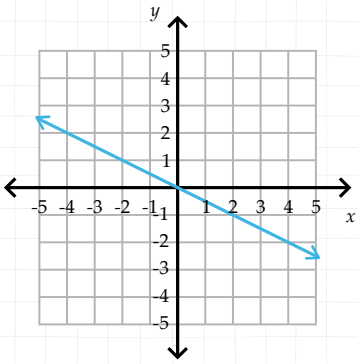
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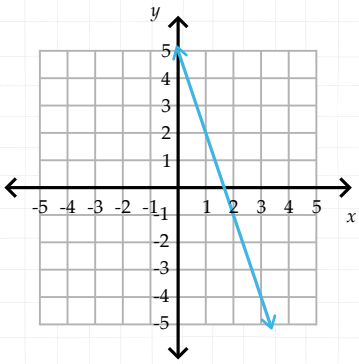
Note that a line with a negative rise and a positive run will always have a negative slope. For example, a rise of –4 and a run of 3 will produce a slope of $\frac{-4}{3}$, which can be written as $-\frac{4}{3}$. Since the slope is useful as a fraction, never write the slope as a mixed number. If the slope is an improper fraction, like this one, leave it as an improper fraction.

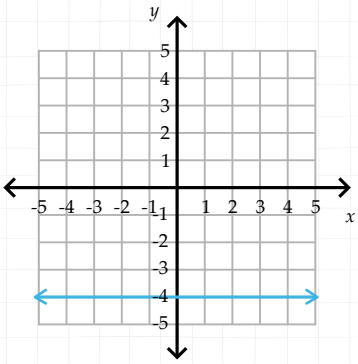
PRACTICE

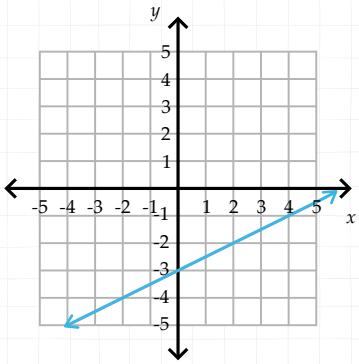
1. Draw a line to match each graph with a slope.









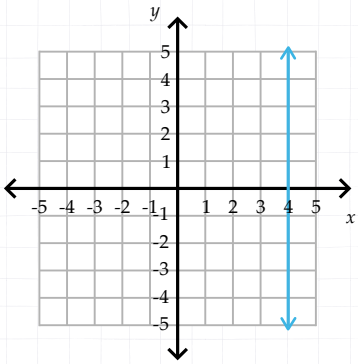


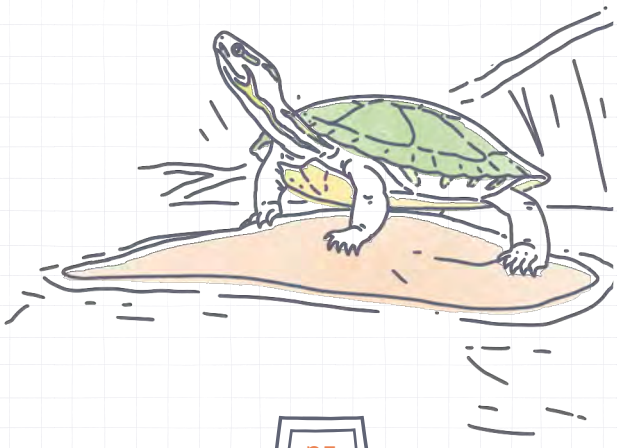
Positive

Negative

Zero

Undefined



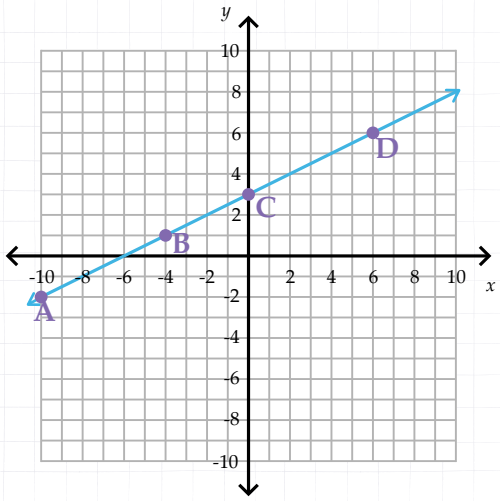


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For each of the graphs below, calculate the slope of the line by following the steps.

✦ Hint: Remember to simplify the slope. The slope between any two points on a line is the same!

2.



a. Draw a vertical line that shows the rise between Points *A* and *B*.

The rise is ____.

b. Draw a horizontal line that shows the run between Points *A* and *B*.

The run is ____.

c. The slope of the line using Points *A* and *B* is ____.

d. Draw a vertical line that shows the rise between Points *C* and *D*.

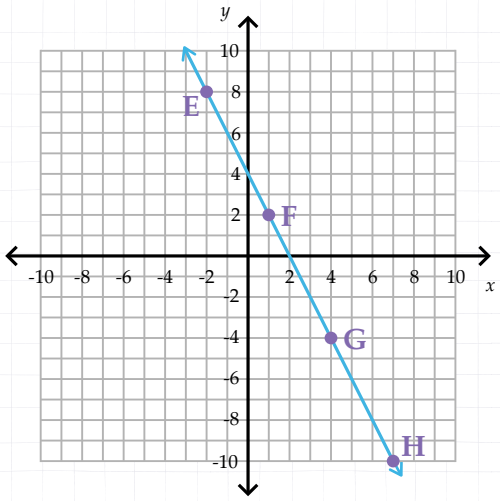
The rise is ____.

e. Draw a horizontal line that shows the run between Points *C* and *D*.

The run is ____.

f. The slope of the line using Points *C* and *D* is ____.

3.



a. Draw a vertical line that shows the rise between Points *E* and *H*.

The rise is ____.

b. Draw a horizontal line that shows the run between Points *E* and *H*.

The run is ____.

c. The slope of the line using Points *E* and *H* is ____.

d. Draw a vertical line that shows the rise between Points *F* and *G*.

The rise is ____ .

e. Draw a horizontal line that shows the run between Points *F* and *G*.

The run is ____.

f. The slope of the line using Points *F* and *G* is ____.

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4. Use the clues to fill in the blanks and find the missing words in the word search below. If you need help, refer to the Lesson Overview.

a. The vertical change in y-values is the _____.

b. The _____ is the horizontal change in x-values.

c. The steepness and direction of the line are given by its _____. It is defined as the change in y-values divided by the change in x-values.

d. A _____ slope shows y-values decreasing as x-values increase.

e. A _____ slope shows y-values increasing as x-values increase.

f. A horizontal line has a slope equal to _____.

g. A _____ line has a slope that is undefined.

V	J	N	U	L	M	M	X	X	N
Z	E	A	E	R	S	L	O	P	E
E	H	R	R	G	U	W	X	P	N
R	X	P	T	I	A	K	M	F	T
O	R	A	R	I	S	T	H	E	M
A	U	H	N	H	C	E	I	C	X
V	N	U	M	S	H	A	C	V	U
Y	Q	X	I	K	H	V	L	M	E
R	W	O	H	V	T	X	K	R	Y
J	O	P	O	S	I	T	I	V	E





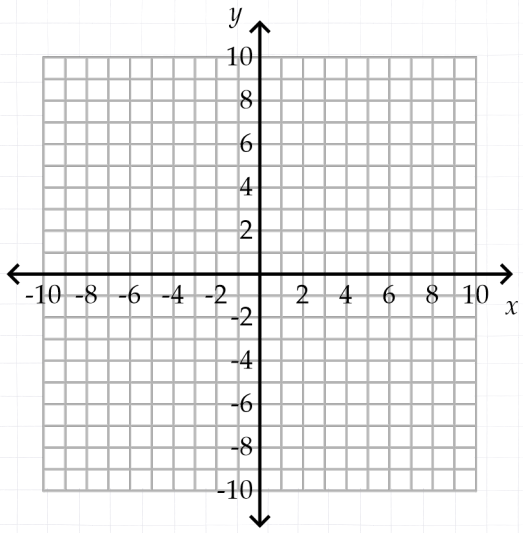
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REVIEW

1. Complete the T-chart by substituting the given x-values in the equation. Then use the ordered pairs from the T-chart to graph the line that represents the equation. L65

$y = 3x - 2$

x	y
–2	
0	
2	
4	



2. Janessa buys a ski coat with an original price of $112. The coat is marked 60% off, and sales tax in Janessa's town is 7%. Find the sale price and total cost of the coat. L47, L48

Sale price: _____ Total cost: _____

3. Debra is creating a scale drawing of her bedroom using a scale of 1 in : 1 ft. Her bed is 42 inches wide and 78 inches long. What should the width and length of her bed be on her scale drawing? L54, L61

✦ Hint: Convert the dimensions of the bed to feet first.

Width: _____ Length: _____

4. Mentally find 25% of each number by dividing by 4.

✦ Tip: Sometimes dividing by 4 is easiest to complete mentally by dividing by 2 twice.

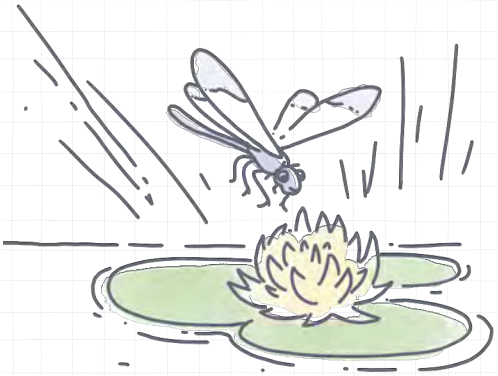
a. 440 _____

b. 72 _____

c. 1,400 _____

d. 36 _____

e. 900 _____



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Graphing Functions

WARM-UP

MENTAL MATH: Complete the problems below mentally.

1. Write the integer that represents each phrase.

   a. A drop of 12 degrees _____ b. A deposit of $50 _____ c. 450 feet below sea level _____

2. Complete each problem.

   a. $5 \bullet (-2) \bullet 3 \bullet (-4) =$ _____ b. $-3 \bullet 4 \bullet (-3) \bullet (-2) =$ _____

LESSON

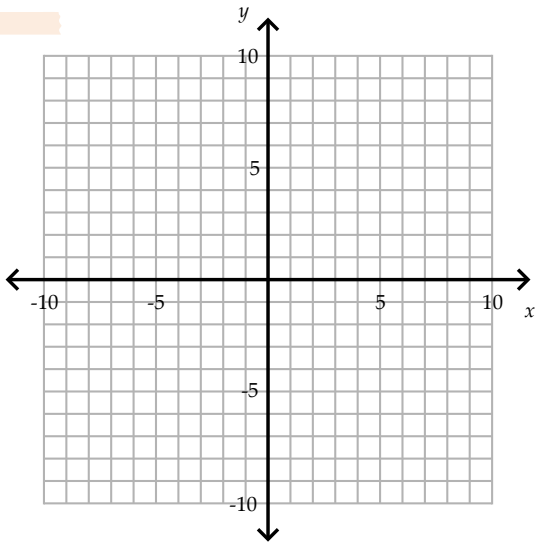
Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Notes section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

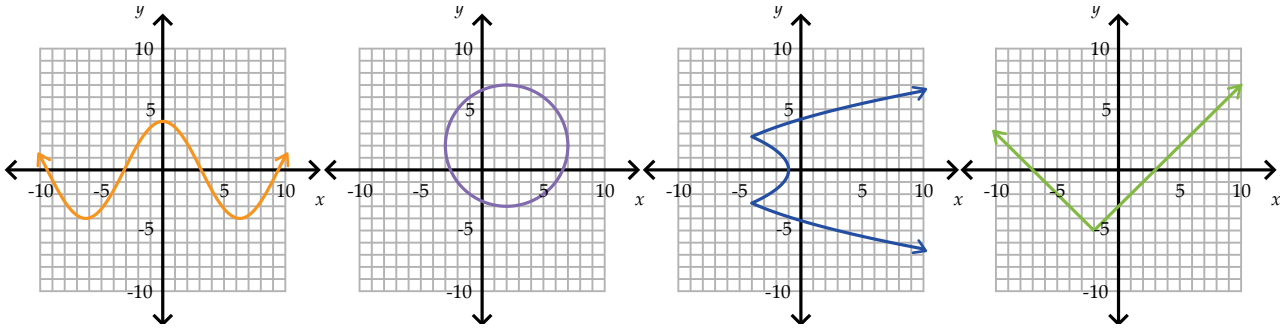
App Video

VIDEO NOTES

$y = x^2 - 3$

x	y	
–3	6	$(-3,6)$
–2	1	$(-2,1)$
–1	–2	$(-1,-2)$
0		(____, ____)
1		(____, ____)
2		(____, ____)
3		(____, ____)





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LESSON OVERVIEW

Nonlinear Functions

When the graph of a function is not a straight line, the function is considered nonlinear. Nonlinear functions can be graphed with a T-chart just like linear functions. Input/output tables can form curves when graphed.

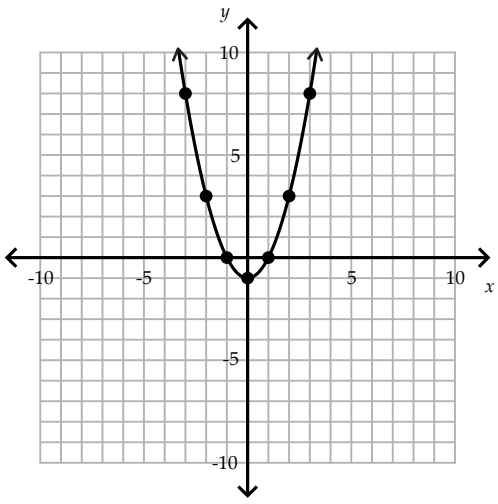
Substitute x-values into the function and perform operations to find the corresponding y-values. Graph the ordered pairs created by x and y. Connect the points to represent the function on the graph. If the points do not form a straight line, draw a curved line as the points are connected. Draw arrows at the ends to show that the relationship between x and y continues.

Example 1: Graph $y = x^2 - 1$.

Substitute the given values of x into the equation and solve for y. Then graph the x- and y-values as ordered pairs. Connect with a curve.

x	y
–3	8
–2	3
–1	0
0	–1
1	0
2	3
3	8

When $x = -3$, $y = (-3)^2 - 1 = 9 - 1 = 8$.
When $x = -2$, $y = (-2)^2 - 1 = 4 - 1 = 3$.
When $x = -1$, $y = (-1)^2 - 1 = 1 - 1 = 0$.
When $x = 0$, $y = 0^2 - 1 = -1$.
When $x = 1$, $y = 1^2 - 1 = 0$.
When $x = 2$, $y = 2^2 - 1 = 3$.
When $x = 3$, $y = 3^2 - 1 = 8$.

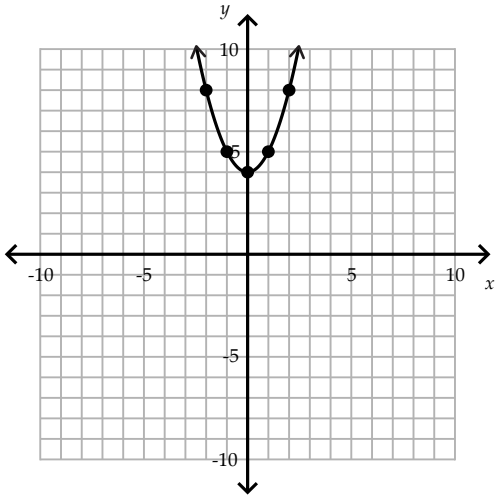


Example 2: Graph $y = x^2 + 4$.

Substitute the given values of x into the equation and solve for y. Then graph the x- and y-values as ordered pairs. Connect with a curve.

x	y
–2	8
–1	5
0	4
1	5
2	8

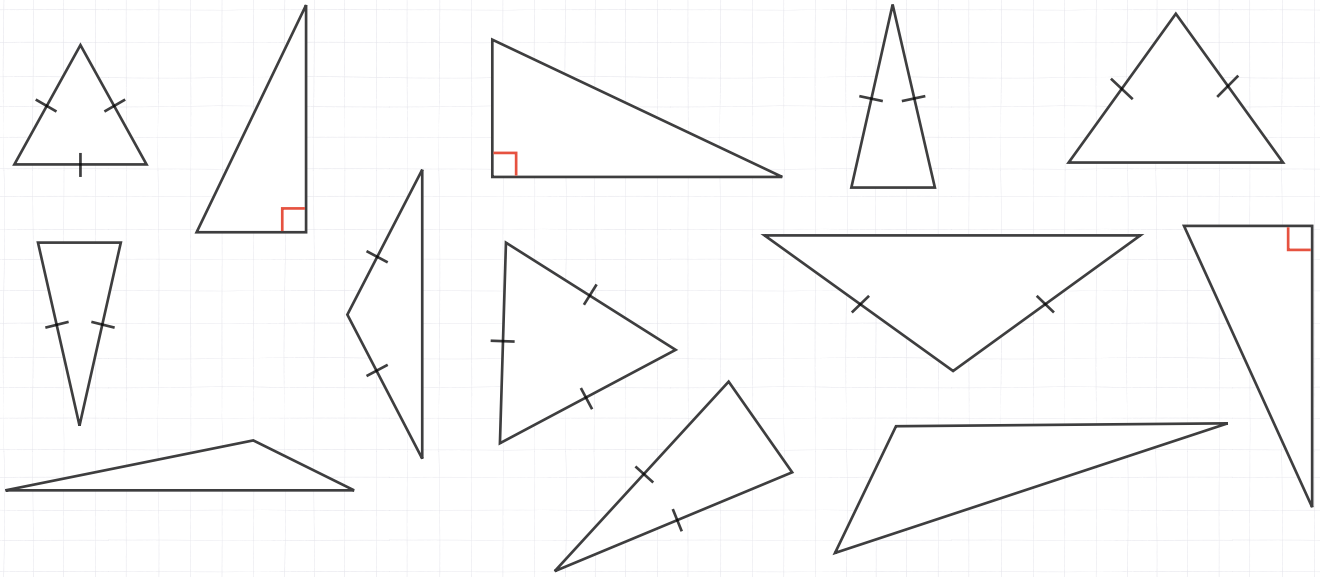
When $x = -2$, $y = (-2)^2 + 4 = 4 + 4 = 8$.
When $x = -1$, $y = (-1)^2 + 4 = 1 + 4 = 5$.
When $x = 0$, $y = 0^2 + 4 = 0 + 4 = 4$.
When $x = 1$, $y = 1^2 + 4 = 1 + 4 = 5$.
When $x = 2$, $y = 2^2 + 4 = 4 + 4 = 8$.



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PRACTICE

1. Color all scalene triangles blue, all isosceles triangles orange, and all equilateral triangles green. Then circle all right triangles, box in all obtuse triangles, and cross out all acute triangles.



2. Color each triangle description and the missing interior angle(s) with the same color. An example is shown.

A triangle has two angles measuring 40°. What is the measure of the third angle?	74°	A right triangle has an angle measuring 36°. What is the measure of the third angle?	80°
60°	What are each of the angle measures in an equilateral triangle?	An isosceles triangle has one angle that measures 20°. What is the measure of each of the two congruent angles?	What is the measure of each of the non-right angles of an isosceles right triangle?
45°	A triangle has two angles measuring 35° and 71°. What is the third angle measure?	100°	54°

3. Recall that in every triangle, the sum of ANY two side lengths must be greater than the length of the third side. Determine if the following side lengths form a triangle. Write *yes* or *no* on the line. If the side lengths do not form a triangle, give the sides that do not meet the criteria. An example is given.

Example: 4, 9, 4 no

4 + 4 is not greater than the third side of 9.

a. 3, 4, 5 _____

b. 3, 5, 3 _____

c. 4, 6, 11 _____

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Figures can be rotated around any point. The coordinate plane below shows a heart being rotated 90° counterclockwise around the point $(8,8)$. It can help to see the rotation by placing the tip of a pencil on the point $(8,8)$ and rotating the paper.

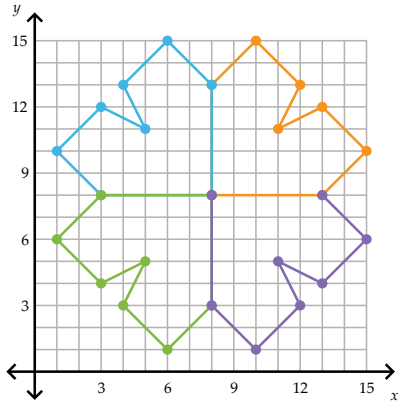
The orange heart is the preimage.

The blue heart shows a 90° counterclockwise turn around the point.

The green heart shows a 180° counterclockwise turn around the point.

The purple heart shows a 270° counterclockwise turn around the point.

A 360° turn around the point would result in the original orange heart.



PRACTICE

1. For each image, draw all the lines of symmetry. Write the number of lines of symmetry in the blank.

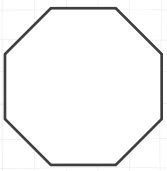
a. Number of lines of symmetry: _____



b. Number of lines of symmetry: _____



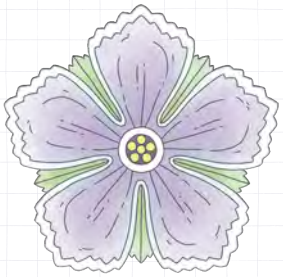
c. Number of lines of symmetry: _____



2. For each figure, find the order of rotational symmetry.

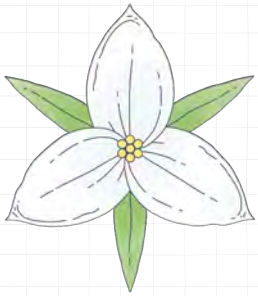
✦ Hint: Find the number of times a shape can be rotated and look the same as the original orientation.

a.



Order of rotational symmetry: ____

b.



Order of rotational symmetry: ____

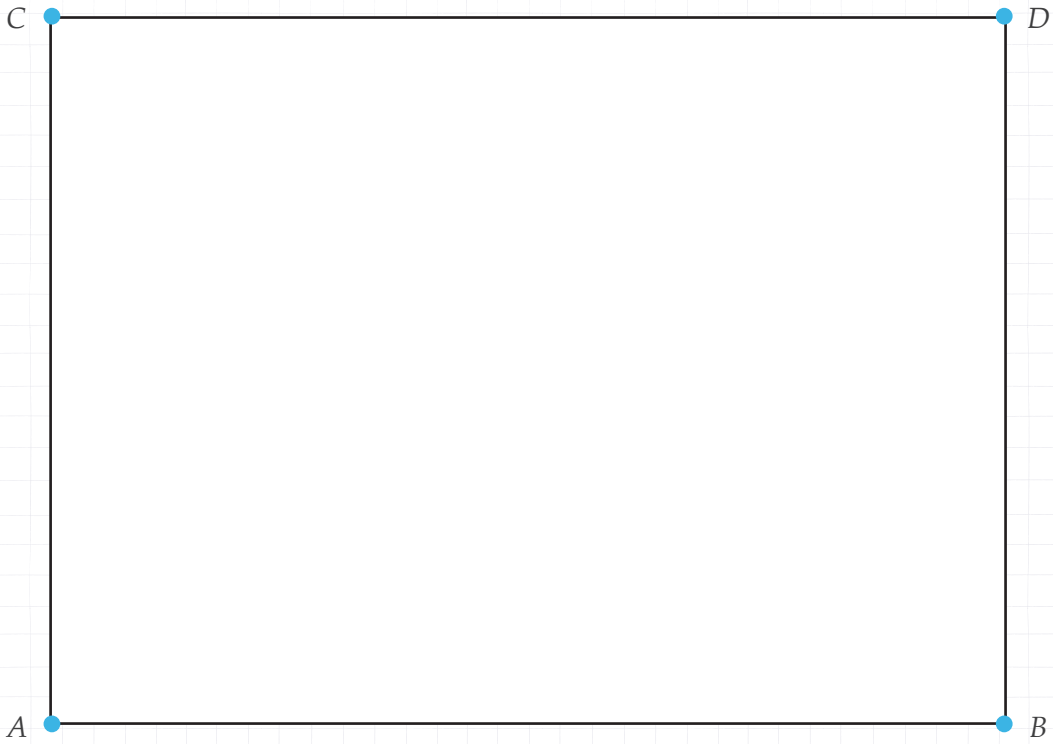
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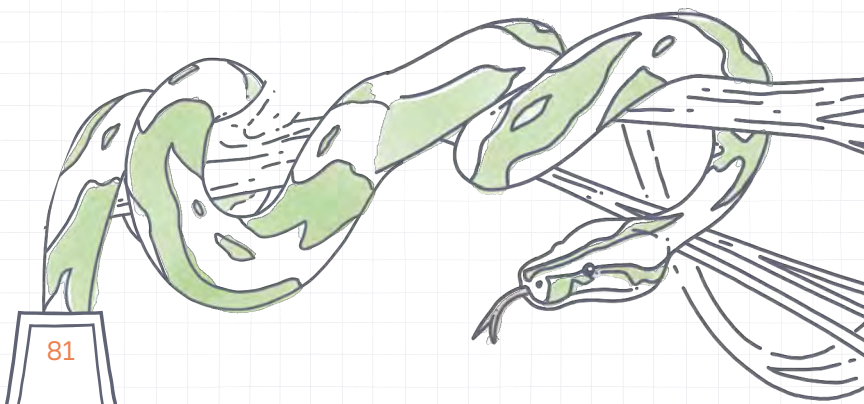
3. Follow the instructions to create a design on the rectangle below.

Note: Erase your compass arcs after bisecting any segments or angles so the image doesn't get too cluttered.

a. Bisect line segment $\overline{AB}$. Label the point where the bisector intersects $\overline{AB}$ as E.
b. Use a straightedge to draw a line segment from E to C and from E to D.
c. Bisect $\angle ECD$. Draw the bisector long enough that it touches the edge of the rectangle.
d. Bisect $\angle EDC$. Draw the bisector long enough that it touches the edge of the rectangle.
e. Label the point where the two bisectors from Parts C and D cross each other as F.
f. Use a straightedge to draw a line segment from E to F.
g. Bisect $\overline{EF}$ and label the point where the bisector intersects $\overline{EF}$ as G.
h. Use a straightedge to draw a line segment from G to A and from G to B.

Now color in the design you created!





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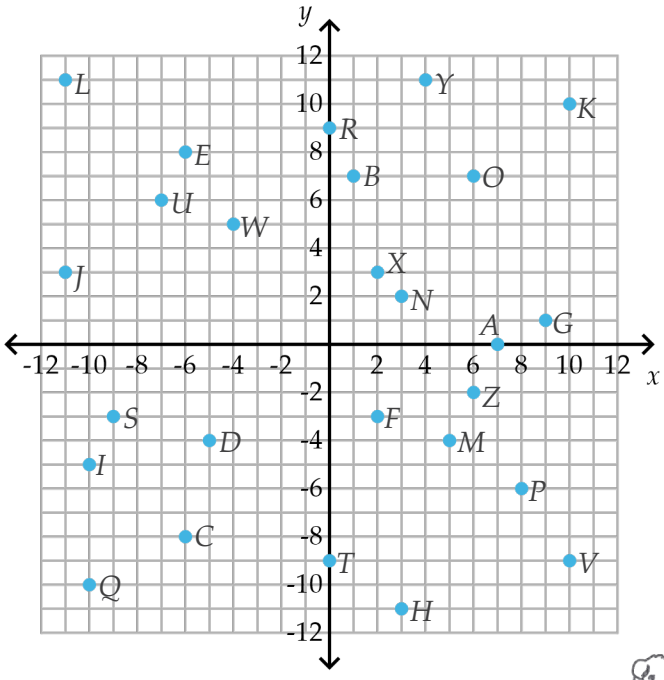
Logic Lesson 3

Spring is a busy season on a farm! Baby animals are born, fields and gardens are prepared, and crops are planted. Barns, stalls, coops, and other animal homes are built or repaired. These tasks can be enjoyable, but they can also require thinking outside the box—and so will the puzzles you complete today! There is no video or review for this lesson.

Who's New on the Farm?

Babies of different animals have special names. For example, a baby goat is called a kid, and a baby cow is called a calf. Use the coordinate plane and the instructions below to decipher the names of other baby farm animals.

Instructions: Locate the letter using the given coordinate. Then go BACKWARD three letters in the alphabet and write that letter on the blank above the given coordinate. Think of the alphabet as a cycle, with *A* following *Z*. For example, if the coordinate (1,7) is given, locate the letter at that point. The letter is *B*. Now go backward three letters in the alphabet. The letter *A* is one letter back, the letter *Z* is two letters back, and the letter *Y* is three letters back. Write the letter *Y* on the blank above the coordinate (1,7).



a. A baby turkey of either gender is called a

$\overline{(-9,-3)}$ $\overline{(0,9)}$ $\overline{(2,3)}$ $\overline{(6,7)}$ $\overline{(-4,5)}$

but a male can be called a

$\overline{(5,-4)}$ $\overline{(-5,-4)}$ $\overline{(3,2)}$ $\overline{(3,-11)}$

and a female can be called a

$\overline{(5,-4)}$ $\overline{(3,-11)}$ $\overline{(-10,-10)}$ $\overline{(-10,-10)}$ $\overline{(1,7)}$.

b. A baby llama or alpaca is called a

$\overline{(2,-3)}$ $\overline{(-7,6)}$ $\overline{(-11,11)}$ $\overline{(-5,-4)}$.

c. A baby horse of either gender is called a

$\overline{(-10,-5)}$ $\overline{(0,9)}$ $\overline{(-5,-4)}$ $\overline{(6,7)}$



but a male can be called a

$\overline{(2,-3)}$ $\overline{(0,9)}$ $\overline{(6,7)}$ $\overline{(-4,5)}$

and a female can be called a

$\overline{(-10,-5)}$ $\overline{(-11,11)}$ $\overline{(6,7)}$ $\overline{(6,7)}$ $\overline{(1,7)}$.

d. The Ugly Duckling was actually a baby swan, or a

$\overline{(2,-3)}$ $\overline{(1,7)}$ $\overline{(-11,3)}$ $\overline{(-10,-10)}$ $\overline{(3,-11)}$ $\overline{(-4,5)}$.

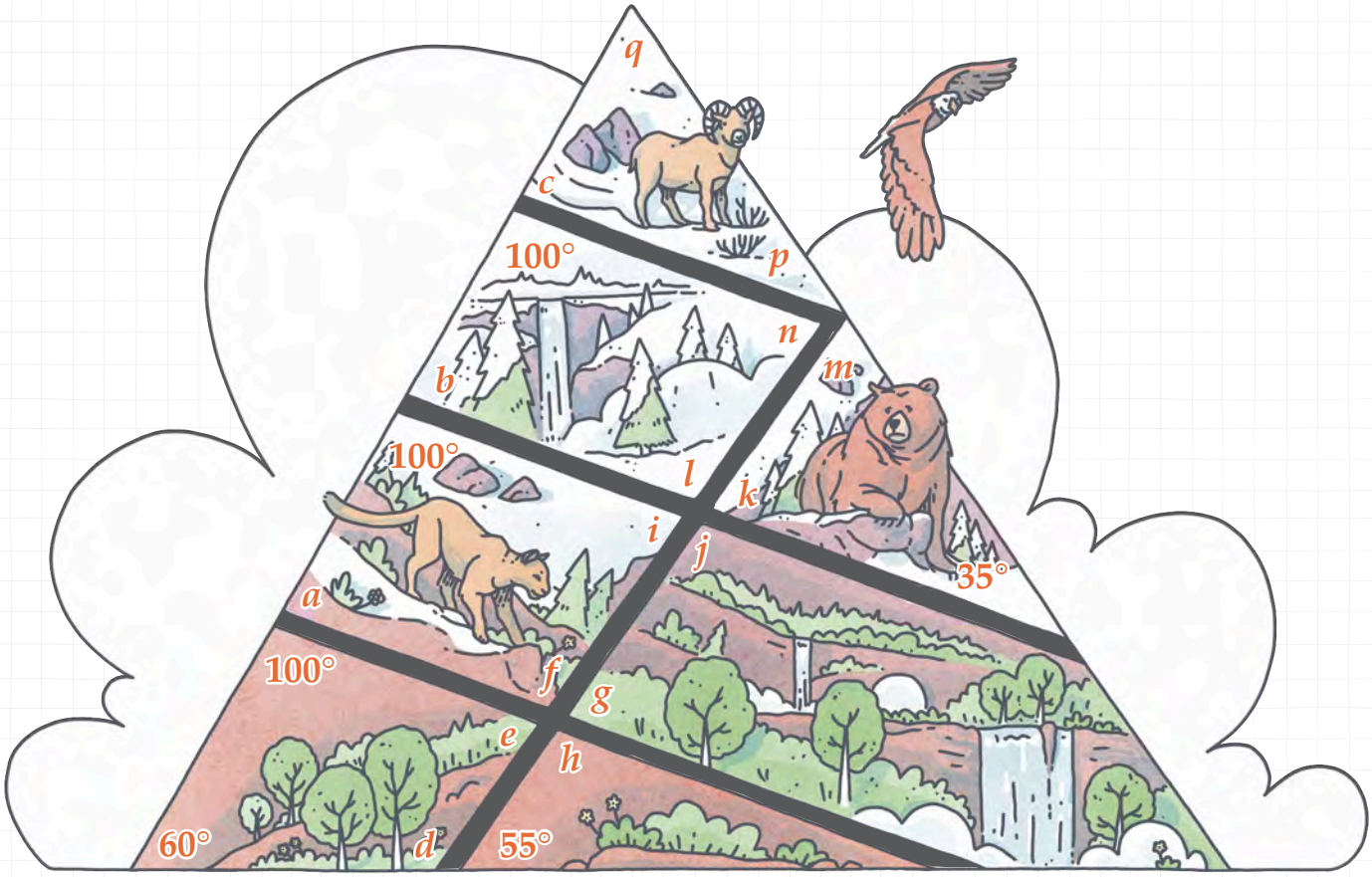
e. Mary had a little lamb. A lamb raised as a pet is sometimes called a



$\overline{(2,-3)}$ $\overline{(0,9)}$ $\overline{(10,-9)}$ $\overline{(10,-9)}$ $\overline{(3,-11)}$ $\overline{(-4,5)}$.

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5. The mountain peak below contains interior angle measures of polygons, complementary and supplementary angles, and vertical angles. Work your way up the mountain to find the measure of the mountain peak!



✦ Hint: The internal angle sum of a triangle is 180°, and the internal angle sum of a quadrilateral is 360°. Some angles may need to be found before others.

$a =$ _____ $b =$ _____ $c =$ _____ $d =$ _____

$e =$ _____ $f =$ _____ $g =$ _____ $h =$ _____

$i =$ _____ $j =$ _____ $k =$ _____ $l =$ _____

$m =$ _____ $n =$ _____ $p =$ _____ $q =$ _____
(measure of the mountain peak)

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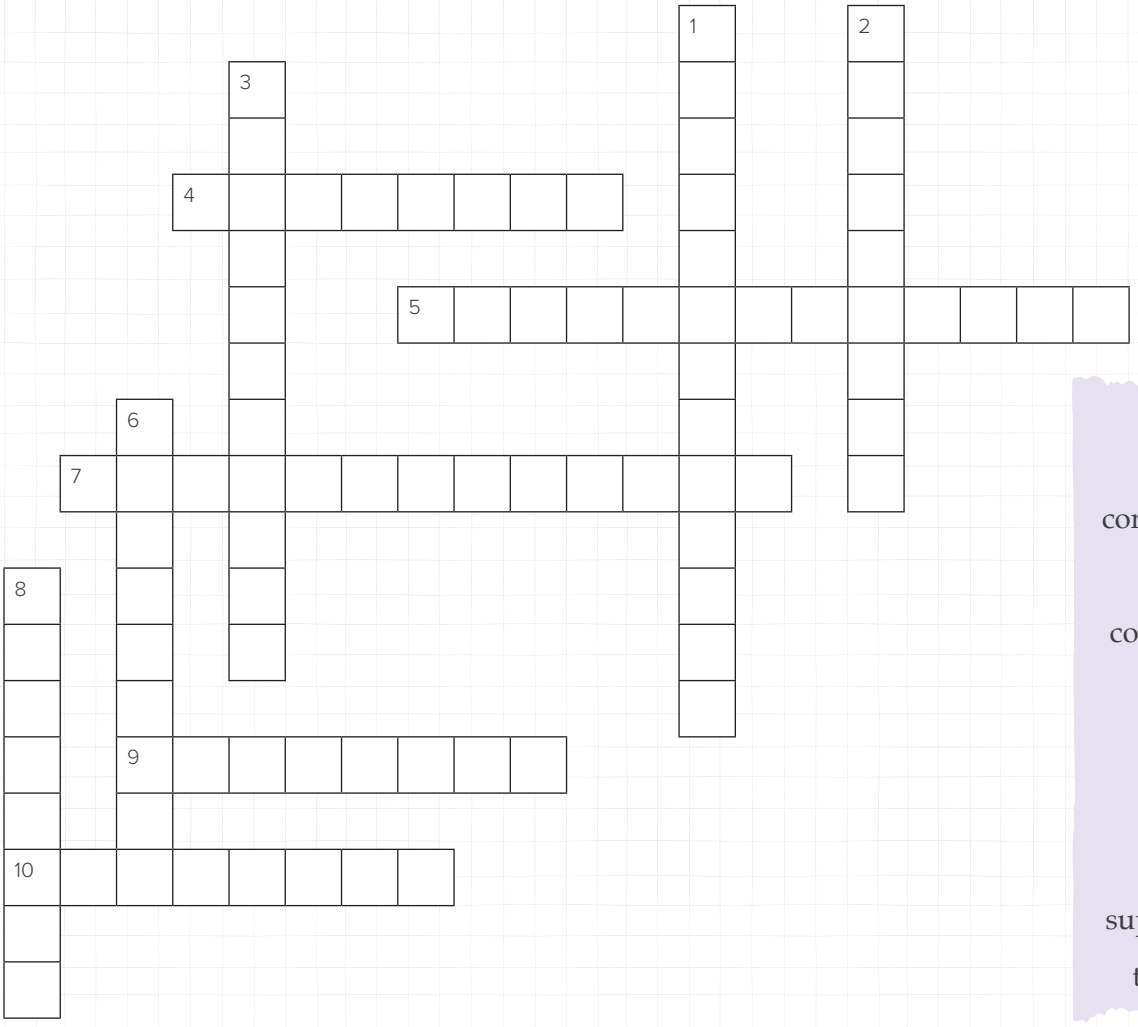
5. Use the hints to solve the crossword puzzle. Use the word bank provided.

DOWN

1. Angles that add to 90° are called _____ angles.
2. Interior angles on opposite sides of the transversal are called _____ interior angles.
3. A _____ is a line that intersects two or more lines.
6. Corresponding angles on parallel lines are _____.
8. Alternate exterior angles are located on _____ sides of the transversal.

ACROSS

4. _____ lines are lines that never intersect and are always the same distance apart.
5. Angles that add to 180° are called _____ angles.
7. Angles that are located in the same position on parallel lines are called _____ angles.
9. Angles that are outside of the parallel lines are called _____ angles.
10. _____ angles are angles that are between the parallel lines.

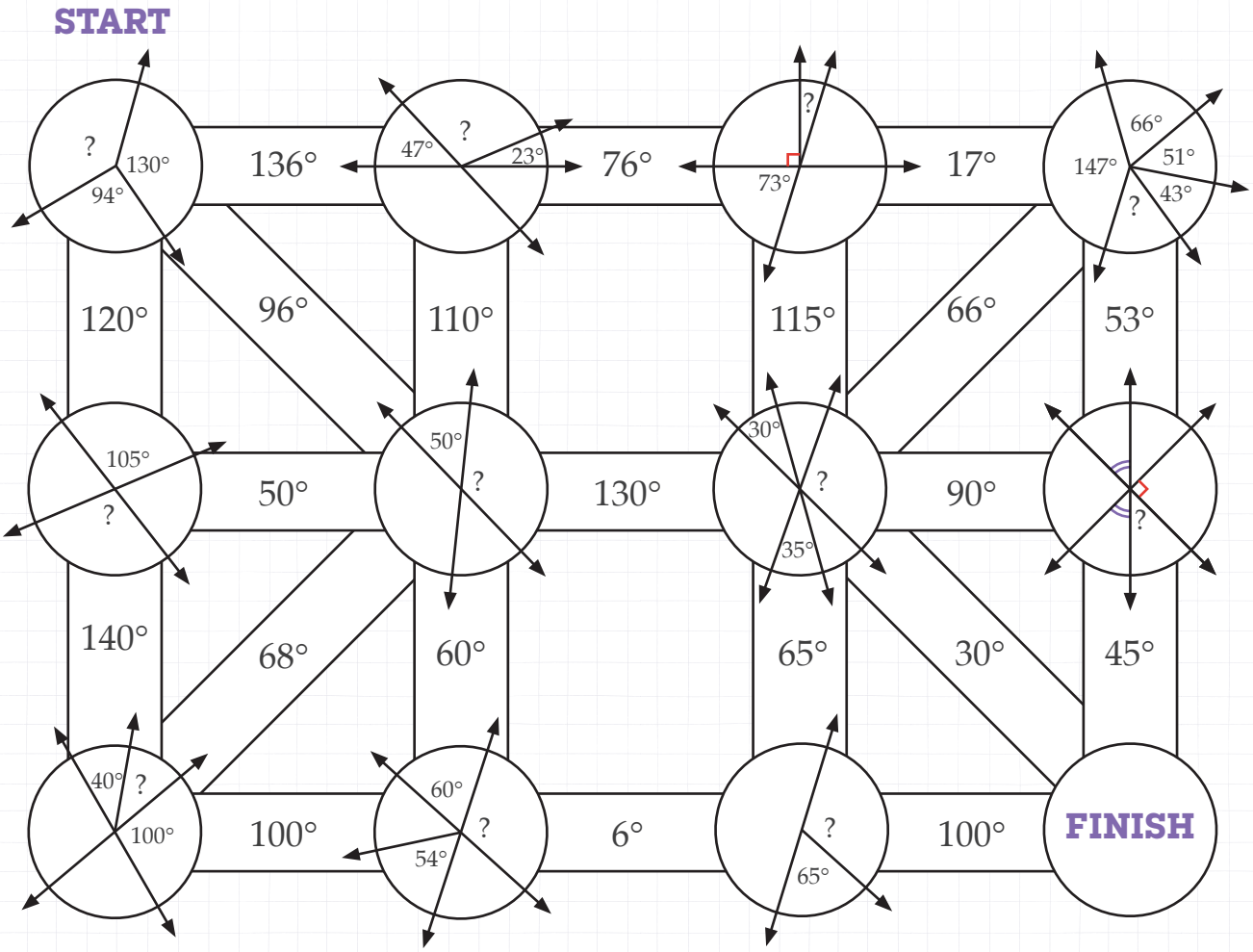


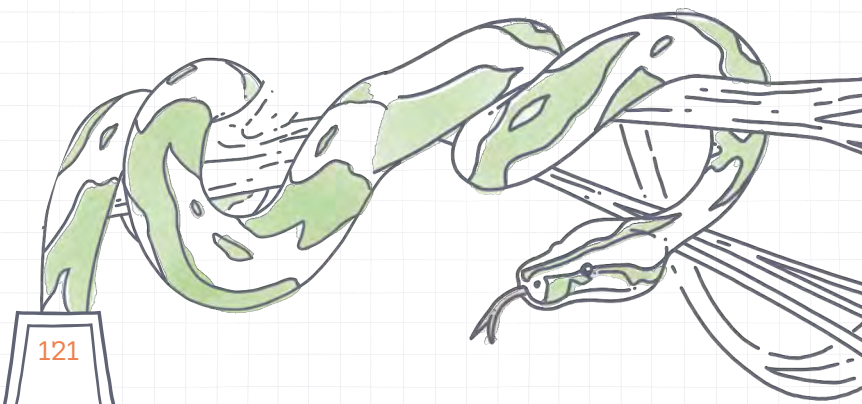
Word Bank

alternate
complementary
congruent
corresponding
exterior
interior
opposite
parallel
supplementary
transversal

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4. Begin at START and find each missing angle measure to make your way through the maze.





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Pythagorean Theorem

SUPPLIES: highlighter

WARM-UP

Evaluate.

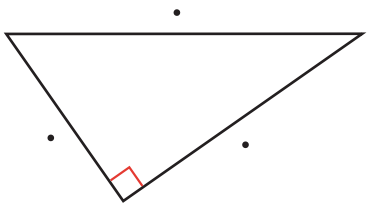
a. $\sqrt{121}$ _____ b. 8^2 _____ c. $\sqrt{169}$ _____

LESSON

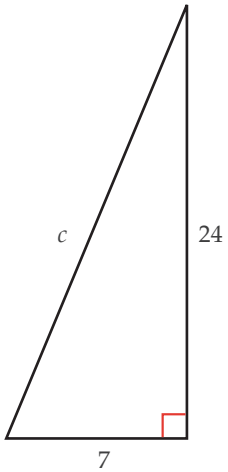
Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Notes section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

App Video

VIDEO NOTES



leg hypotenuse leg



_____ + _____ = _____

_____ + _____ = c^2

_____ + _____ = c^2

_____ = c^2

_____ = c

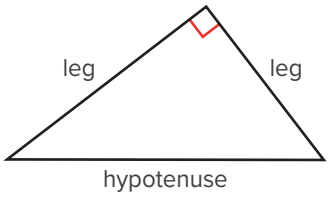
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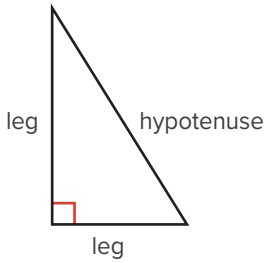
LESSON OVERVIEW

Defining the Pythagorean Theorem

Right triangles appear in things like buildings, art, and navigation. Knowing how to calculate the length of a missing side on a right triangle is a foundational piece of mathematics.

The longest side of a right triangle is called the ***hypotenuse***. The hypotenuse is across from, or opposite, the right angle. The other two sides of a right triangle, the sides adjacent to the right angle, are called the ***legs***. Two right triangles are shown below.



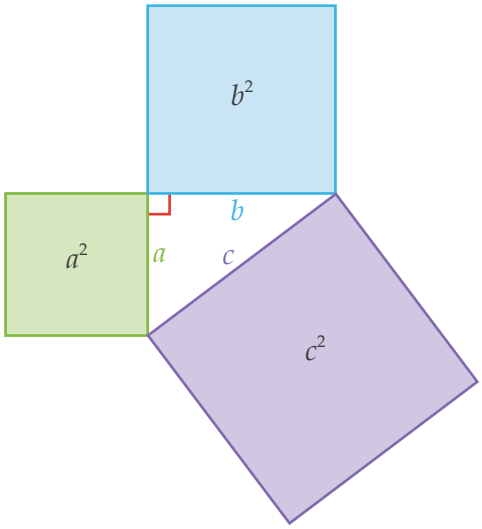


The ***Pythagorean Theorem***, named for the Greek philosopher Pythagoras, states that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two side lengths. This relationship can be represented using this equation:

$$a^2 + b^2 = c^2$$

In the Pythagorean Theorem, a and b represent the lengths of the legs, and c represents the length of the hypotenuse of a right triangle.

The Pythagorean Theorem can be thought of as representing the area of two squares having side lengths a and b being added to equal the area of a third square with side length c. The areas of squares a^2 and b^2 add to equal the area of the third square, c^2.



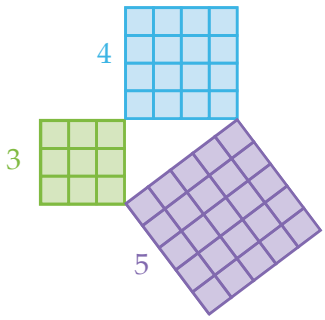
To illustrate this, let $a = 3$, $b = 4$, and $c = 5$.

Squaring each value gives the area of each square.

$a^2 = 3^2 = 9$ $\quad$ $b^2 = 4^2 = 16$ $\quad$ $c^2 = 5^2 = 25$

The sum of the two smaller areas is equal to the third area. $9 + 16 = 25$

Therefore, $a^2 + b^2 = c^2$.

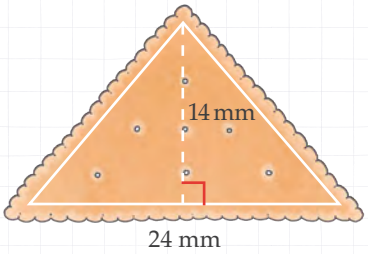


Sets of integers that satisfy the Pythagorean Theorem are called ***Pythagorean triples***. The smallest integers that form a Pythagorean triple are 3, 4, and 5, as shown above.

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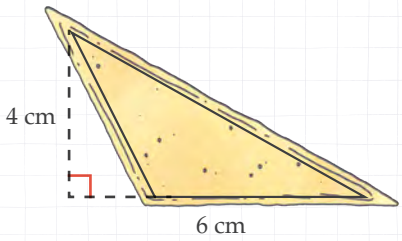
2. After learning about areas of polygons, Margie sat down for lunch and realized that much of her food was shaped like polygons! She decided to find the approximate area of some of her food before eating it. What is the area of each shape shown below? Be sure to include units.

a.



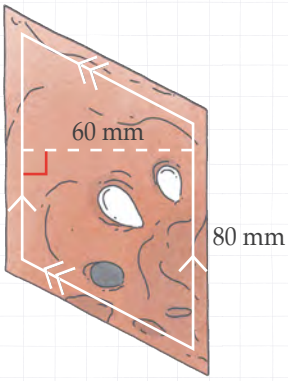
The area of the shape shown on the cracker is _____.

b.



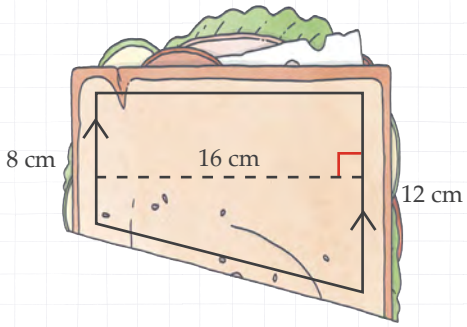
The area of the shape on the tortilla chip is _____.

c.



The area of the shape in the watermelon piece is _____.

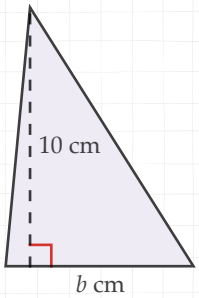
d.



The area of the shape in the sandwich is _____.

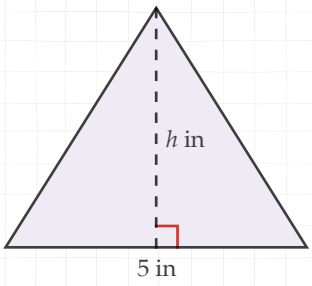
3. Find the missing measurement on each figure.

a. The area of the triangle is 35 cm^2.



The base is _____.

b. The area of the triangle is 10 in^2.



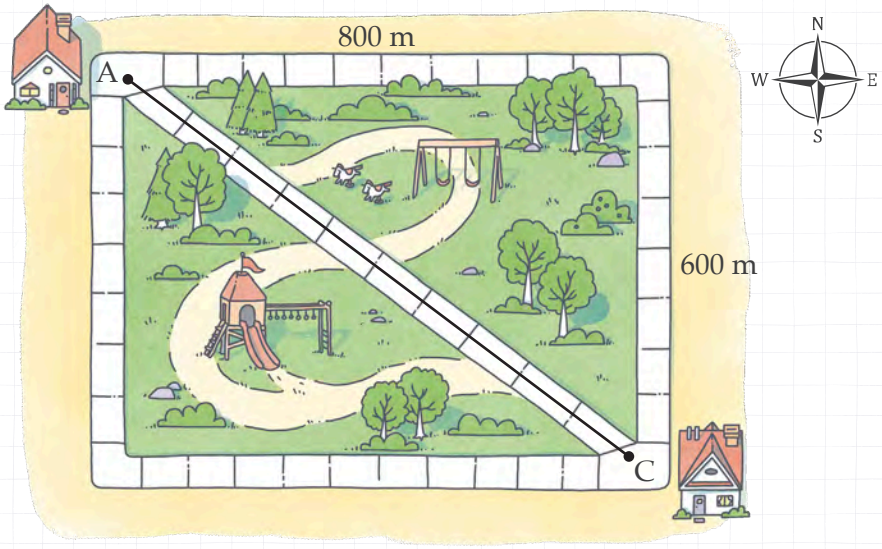
The height is _____.

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REVIEW

Refer to the information and diagram below for all problems in this review.

Jayda lives near the northwest corner (Point A) of the park shown below. Her friend Emily lives near the southeast corner (Point C). There is a sidewalk around the entire perimeter of the park, as well as one across the park connecting Point A to Point C. Jayda and Emily often meet at the park to roller skate on the sidewalks.



1. If Jayda roller skates east and then south around the outside of the park, what distance will she travel to reach Emily's house? L82

_____

2. If Jayda roller skates across the park directly to where Emily is, what distance will she travel? L81

_____

3. Suppose that yesterday Jayda roller skated east and then south to meet Emily (refer to Problem 1), and today she roller skated directly across the park (refer to Problem 2) instead. What is the percent decrease in the distance Jayda traveled to meet Emily? Round to the nearest percent. L48

_____

4. What is the area of the park in square meters? L83

_____

5. Jayda's friend Kevin mows the grass at the park. If $\frac{4}{5}$ of the park is covered with grass, how many square meters of grass does Kevin mow? L43

✦ Hint: Find $\frac{4}{5}$ of the area found in Problem 4.

_____

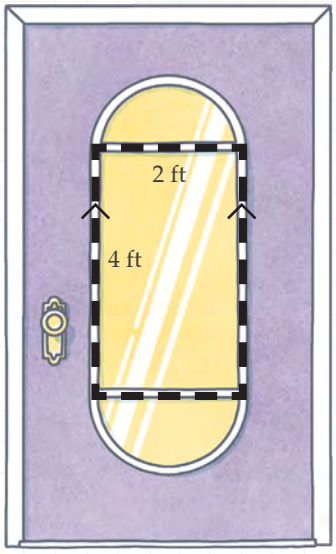
6. Convert the area of the park (found in Problem 4) to square kilometers. L55

_____

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3. A door maker is trying to choose between two different window designs on his doors. Find the approximate perimeter of each window so he knows how much molding each requires. Then find the approximate area of each window to the nearest hundredth so he knows how much light it will let through.

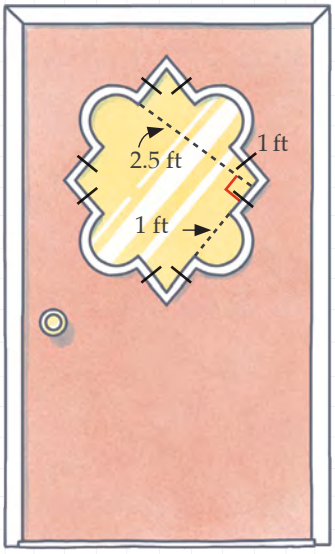
a.



Perimeter: _____

Area: _____

b.



Note: This window is a parallelogram with four semicircles. The height of the parallelogram is given, and the base can be found by adding two sides with tick marks to the diameter of the semicircle.

Perimeter: _____

Area: _____

REVIEW

1. Ruby and Indy are repainting their bedroom walls. Their room is 12 feet long by 13 feet wide, and the walls are 8 feet high.

   a. What is the total area of the walls in the bedroom? L83

   _____

   b. The girls will use two coats of paint on each wall. One gallon of paint covers 400 square feet. How many gallons of paint will Ruby and Indy use? L43

   ✦ Hint: Multiply the area of the walls from Part A by two, since two coats of paint will be used. Then use the fact that one gallon covers 400 square feet to write and solve a proportion to find the answer.

   _____

2. A triangle is translated left three units and down four units on a coordinate plane. The coordinates of the preimage are given. Write the coordinates of the image. L72

Preimage	$(-1,1)$	$(4,8)$	$(3,-5)$
Image			

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Mixed Review

This lesson is a mixed review. There is no video, practice, or review section.

SHARPEN YOUR SKILLS!

Functions can be represented in multiple ways. A function can be represented as an equation, a graph, or a table. Each representation can emphasize different features of the function. Seeing all the representations together is helpful in becoming more familiar with the properties of functions.

Function 1:

Jude is working with his dad over the summer repairing and filling swimming pools. His first job is assisting his dad in filling up a brand-new kiddie pool in their backyard. Water fills up the pool at a rate of 3 gallons per minute.

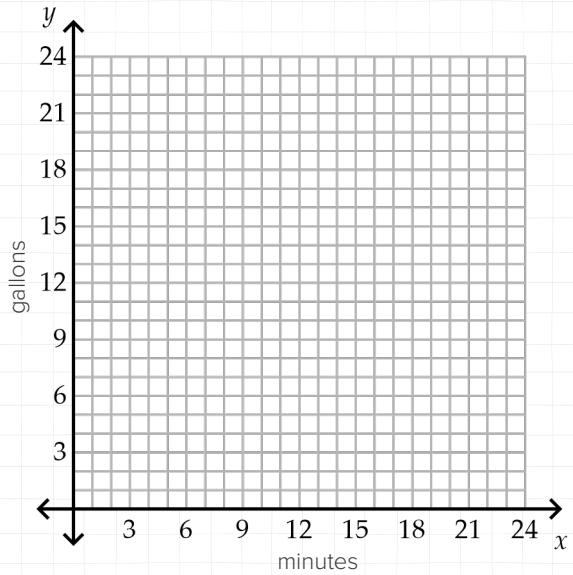
Fill in the y-values in the table by multiplying each x-value by 3. Two examples are given.

x (minutes)	y (gallons)
0	0
1	3
2	_____
3	_____
4	_____
5	_____
6	_____
7	_____

Now write an equation in slope-intercept form that represents the amount of water in the kiddie pool as time passes. The rule is "Multiply the input by 3." Then identify the slope and y-intercept.

$y =$ _____ $m =$ _____ y-intercept: (___, ___)

Draw a graph of the linear function.



Notice that the graph shows only the first quadrant. This is because including other quadrants would include negative numbers. Since it doesn't make sense to have negative time or a negative amount of water in the pool, it doesn't make sense to include the other quadrants in the graph.

Also notice that the graph has a positive slope. As the x-values increase, the y-values increase. That means as the number of minutes increases, the number of gallons increases.



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UNIT 3 | LESSON 88

Unit 3 Review

Complete this Unit Review to prepare for the Unit Assessment. There is no video, lesson, or practice. Because Unit Reviews include practice for an entire unit, they may take longer than regular lessons, and students may decide to take two days to finish.

SUPPLIES: compass, protractor

Spring Cleaning

Spring cleaning and preparations are underway in the Reid household in Auckland, New Zealand.

Scale Drawings
Lesson 61

1. Mateo is mapping out the family garden. The garden plot is 45 meters long and 30 meters wide. His map of the garden is 15 centimeters long.

   a. What is the scale for his map?

   _____

   b. How wide is the garden on his map?

   _____

Direct and Inverse Proportions
Lessons 62–64

2. Aria works with Mateo to figure out how many kiwi vines they need. They know that each mature kiwi vine produces about 20 kg of fruit.

   a. Do kiwi vines and the fruit produced represent a direct or inverse proportion? Why?

   direct / *inverse* (Circle one.)

   _____

   _____

   _____

   b. How many vines should they plant if they want 100 kg of fruit once the plants mature?

   _____

   c. The amount of time it takes to prepare the garden plot decreases as the number of people working it increases. Assuming that everyone works at the same pace, does this represent a direct or inverse proportion? Why?

   direct / *inverse* (Circle one.)

   _____

   _____

   _____

3. Each room in the house takes approximately 3 hours to fully clean.

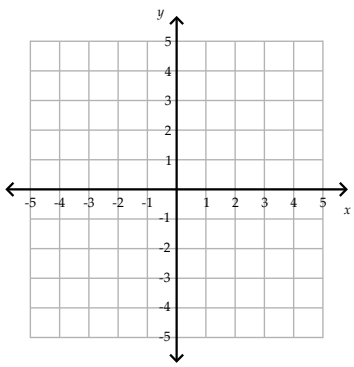
   a. If x is the number of rooms in the house and y is the amount of time spent cleaning, what is the constant of proportionality for this direct proportion?

   $k =$ _____

   b. Write an equation to show the relationship between x and y.

   _____

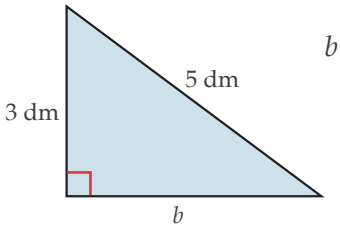
   c. Use the proportionality constant to draw a graph that represents this directly proportional relationship.



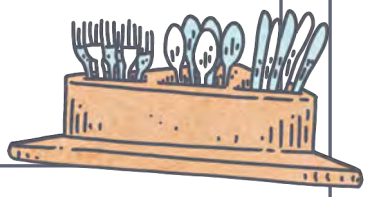
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Pythagorean Theorem
Lesson 81

12. While cleaning out the kitchen, Mr. Reid decides to build a triangular corner shelf to hold some of their kitchen utensils. Use the Pythagorean Theorem to solve for the missing side length.



$b =$ _____

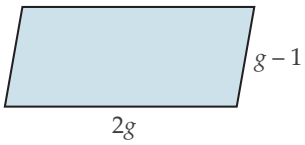


Perimeter and Area
Lessons 82–85

Back in the garden, Mrs. Reid is considering how to arrange the plots for two berry plants that are native to New Zealand: *kōtukutuku* and *tātarāmoa*.

13. The perimeter of the parallelogram-shaped garden plot is 46 m. Find the length and width of the plot.

✦ Hint: The value of g is not a side length but must be found first.

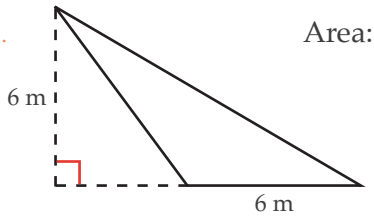


Length: _____ Width: _____

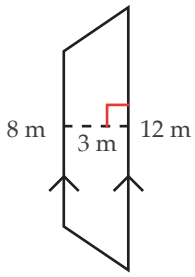


14. Mrs. Reid also wants to put two new plots in her garden for *horopito* and *kawakawa*, which are both used for traditional Māori medicines. Find the area of each plot below.

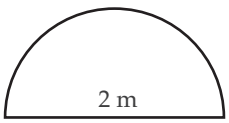
a. Area: _____



b. Area: _____

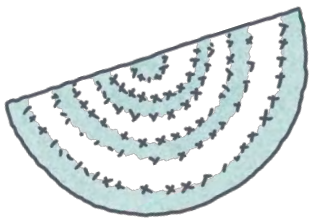


15. New Zealand is known for the high-quality wool it produces, which is extremely soft and also sustainable. Mrs. Reid is creating wool rugs for her home. Find the perimeter and area of the rug shown below, rounding to the nearest hundredth.



Area: _____

Perimeter: _____



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Unit 3 Assessment

SUPPLIES: ruler, protractor, compass

- This assessment covers concepts taught in Unit 3. Problems are designed to assess multiple skills. Read the instructions carefully and do not rush through the problems.
- You may use the Reference Chart at the back of the book for the assessment. Calculators should be used only when noted.
- Lesson numbers are given by each problem so you can review lessons for any answers that are incorrect.

1. On a map, Los Angeles and New York City are 21 cm apart as the crow flies. The scale on the map is 1 cm : 117 mi. How many miles apart are Los Angeles and New York City? L61

_____

2. For each table determine if x and y form a direct or inverse proportion. Then find k. L62, L63

a.

x	3	8	10	14
y	18	48	60	84

direct / *inverse* (Circle one.)

$k =$ _____

b.

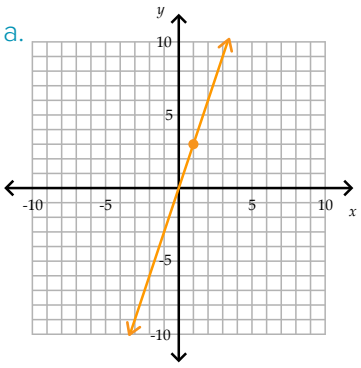
x	1	3	6	11
y	66	22	11	6

direct / *inverse* (Circle one.)

$k =$ _____

3. Determine if each graph is directly proportional. If it is, find k. L64

a.



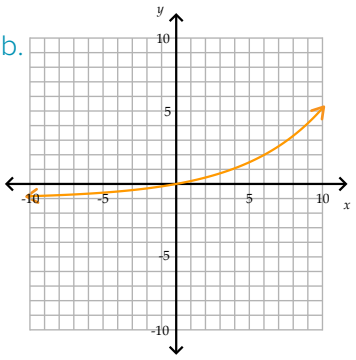
Is it directly proportional?

_____

If yes, find k.

$k =$ _____

b.



Is it directly proportional?

_____

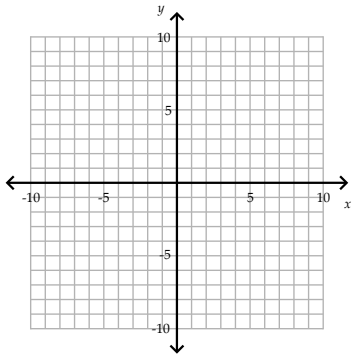
If yes, find k.

$k =$ _____

4. For each given equation, complete the T-chart and use the ordered pairs to graph the equation. L65, L70

a. $y = -3x + 1$

x	y
–2	
–1	
0	
1	
2	



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Enrichment: Circumference and Diameter



This is an enrichment lesson. Mastery is not expected at this level. There is no video, practice, or review. A calculator may be used for this entire lesson.

Part 1: Exploring Circumference and Diameter

SUPPLIES: long piece of string, ruler or yardstick, marker or tape

In Part 1 you will explore the circumference and diameter of objects around your home. The ***diameter*** of a circle is the distance across a circle through the center. The ***circumference*** of a circle is the distance around a circle.

Step 1: Gathering Items

Find three items in your home that are shaped like circles and can be measured. Write the name of each item on the lines on the next page. Suggestions of items are given below:

toilet paper roll	jar or jar lid	clock	ring
circular window	bracelet	flying disk	mirror
water bottle	bucket	bowl	cup
food container	vase	roll of tape	can of food

Step 2: Finding the Circumference

a. Wrap the string around the edge of your first circular item.

b. Use a marker or a small piece of tape to mark the spot where the string meets and overlaps. This will represent the distance around your circular item.

c. Line the end of the string up with 0 on the ruler or yardstick and measure the length of the string to where you marked it to find the circumference.

d. Record the circumference of your first object to the nearest quarter of an inch. Write this measurement as a decimal number.

Step 3: Finding the Diameter

a. Use the ruler or yardstick to measure the diameter of your object. Be sure you measure across the center of the circle.

b. Record the diameter of your object to the nearest quarter of an inch. Write this measurement as a decimal number.

Step 4: Finding the Ratio of Circumference to Diameter

a. Write the ratio of circumference to diameter as a fraction using your measurements.

b. Use a calculator to divide the ratio, $C \div d$, and write the quotient rounded to the ten thousandths place (four decimal places).

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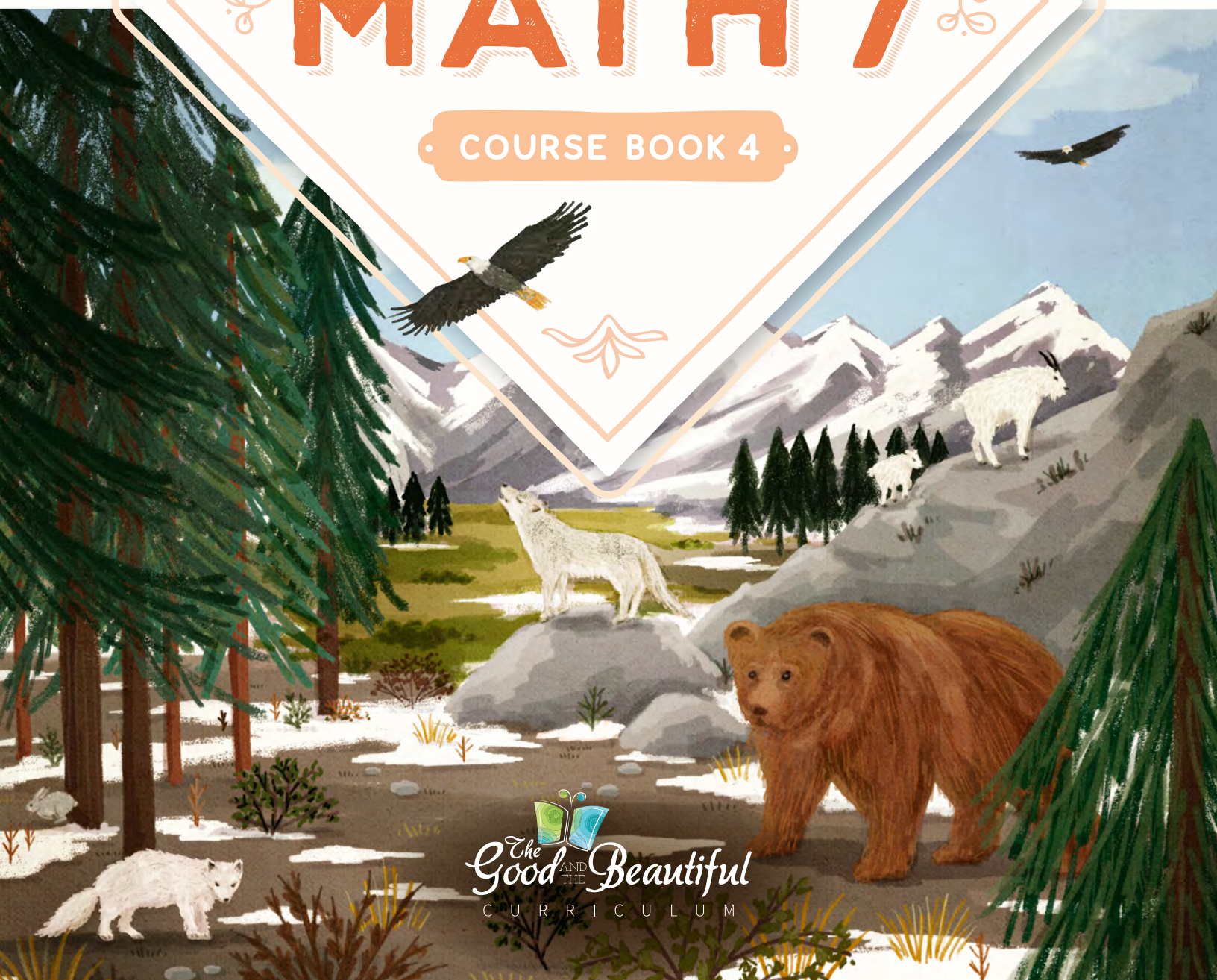
LESSONS
91–120

UNIT
4

Simply Good and Beautiful

MATH 7

COURSE BOOK 4



The Good and the Beautiful
CURRICULUM

COURSE BOOK 4

TABLE OF CONTENTS

UNIT 4 OVERVIEW 1

Lesson 91: Scale Factor with Area 2
Lesson 92: Arcs and Sectors 8
Lesson 93: Geometric Solids 14
Lesson 94: Surface Area of Prisms and Pyramids 21
Lesson 95: Surface Area of Cylinders, Cones, and Spheres 28
Lesson 96: Surface Area of Composite Solids 36
Lesson 97: Volume of Prisms and Cylinders 42
Lesson 98: Volume of Other Geometric Solids 48
Lesson 99: Polynomials 54
Lesson 100: Multiplying Polynomials 59
Lesson 101: Simplifying Rational Expressions 64
Lesson 102: Factoring Polynomials 70
Lesson 103: Populations and Sampling Methods 77
Lesson 104: Data Displays: Part 1 83
Lesson 105: Logic Lesson 4 90
Lesson 106: Measures of Central Tendency 93
Lesson 107: Interpreting Measures of Central Tendency 98
Lesson 108: Data Displays: Part 2 104
Lesson 109: Scatter Plots 111
Lesson 110: Interpreting Graphs 118
Lesson 111: Simple Probability 124
Lesson 112: Types of Events 131
Lesson 113: Sample Space 137
Lesson 114: Compound Probability 143
Lesson 115: Probability Simulation 150
Lesson 116: Unit 4 Review 154
Lesson 117: Course Review 158
Lesson 118: Course Assessment 162
Lesson 119: Enrichment: Patterns with Divisibility 169
Lesson 120: Fun with Graphing 174

Reference Chart 179

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UNIT 4 OVERVIEW

LESSONS 91–120

CONCEPTS COVERED

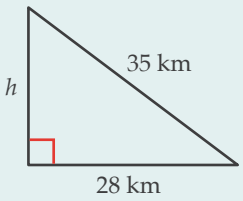
- Bar graphs
- Biased and unbiased samples
- Bimodal and unimodal graphs
- Box plots
- Chords, arcs, sectors, and central angles
- Circle graphs
- Clusters of data
- Complementary events
- Compound probability
- Cross sections of geometric solids
- Determining correlation on graphs
- Experimental probability
- Factoring the GCF from binomials
- Factoring the GCF from trinomials
- Finding a missing data value given the mean
- Finding a sample space
- Finding arc length
- Finding area of sectors
- First, second, and third quartiles
- Frequency tables
- Geometric solids
- Histograms
- Identifying better measures of center
- Identifying modes on a graph
- Independent and dependent events
- Interpreting graphs
- Interpreting measures of central tendency
- Interquartile range
- Line graphs
- Lines of best fit
- Measures of central tendency (mean, median, mode)
- Multiplying monomials by binomials
- Multiplying monomials by monomials
- Mutually exclusive events
- Nets of three-dimensional figures
- Pictographs
- Properties of polyhedra
- Random samples
- Range of data sets
- Sample and sample size
- Scatter plots
- Simple probability
- Simple, stratified, and systematic samples
- Simplifying rational expressions
- Statistics and surveys
- Stem-and-leaf plots
- Surface area of composite solids
- Surface area of cones using a formula
- Surface area of cylinders using nets
- Surface area of prisms using nets
- Surface area of pyramids using nets
- Surface area of spheres using a formula
- Symmetric, right-skewed, and left-skewed graphs
- Theoretical probability
- Understanding outliers
- Volume of cubes and other rectangular prisms
- Volume of cylinders
- Volume of triangular prisms
- Writing monomials as products of factors

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Geometric Solids

WARM-UP

Use the triangle for Parts A and B.



a. Find h.

$h =$ _____

b. Find the area of the triangle.

Area: _____

LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Notes section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

App Video

VIDEO NOTES

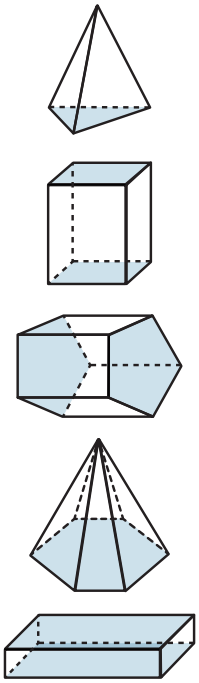
Square

Rectangular

Triangular

Pentagonal

Hexagonal



Prism

Pyramid

Cross section: the _____ formed by cutting straight through a _____

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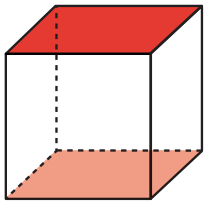
LESSON OVERVIEW

Three-dimensional figures are often referred to as geometric solids, and they have special properties. A ***polyhedron*** (plural: polyhedra) is a three-dimensional figure with polygons as faces. A ***face*** of a polyhedron is a flat surface on the solid. An ***edge*** is a line segment formed where two faces meet, and a ***vertex*** is a point where two or more edges meet. (The plural of vertex is vertices.)

A special type of polyhedron is a prism. A ***prism*** is a solid with two congruent, parallel bases and flat sides. Prisms are often named for the shape of their bases. All non-base faces of a prism are rectangles. Some examples of prisms are below.

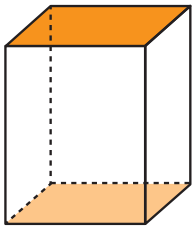
Cube

Each face is a square.



Square Prism

The bases are squares. All other faces are rectangles.



Rectangular Prism

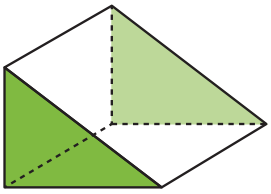
The bases are rectangles. All other faces are also rectangles.



Note: Cubes, square prisms, and rectangular prisms all have 6 faces, 12 edges, and 8 vertices.

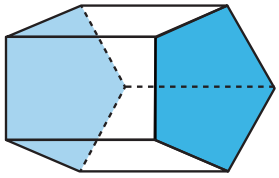
Triangular Prism

The bases are triangles. All other faces are rectangles.



Pentagonal Prism

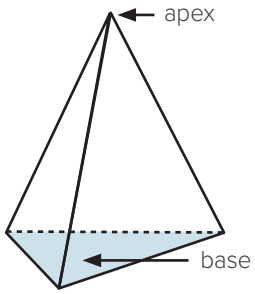
The bases are pentagons. All other faces are rectangles.



Another special type of polyhedron is a pyramid. A ***pyramid*** is a solid with one base and flat sides. A pyramid is often named for the shape of its base. All non-base faces of a pyramid are triangles. The apex of a pyramid is the vertex where all the triangular faces meet.

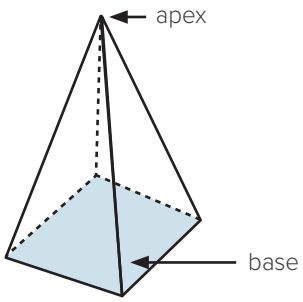
Triangular Pyramid

The base is a triangle. All other faces are triangles.



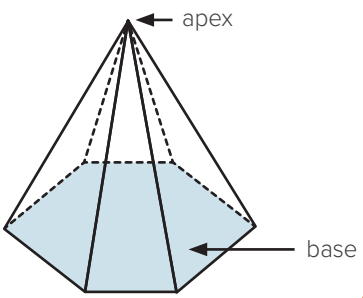
Square Pyramid

The base is a square. All other faces are triangles.



Hexagonal Pyramid

The base is a hexagon. All other faces are triangles.

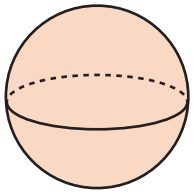


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The ancient Egyptians built square pyramids. Pyramids are also found in art and modern architecture, like the entrance to the Louvre Museum in France.

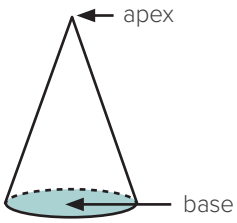
Below are examples of geometric solids that are not polyhedra.

Sphere



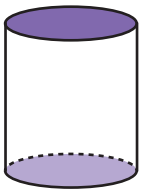
A ***sphere*** is a geometric solid that is not a polyhedron because the surface of a sphere is not made from polygons. Every point on the surface of a sphere is the same distance from the center of the sphere. Examples of spheres include bowling balls and oranges.

Cone



A ***cone*** has a circular base and a curved surface. The tip of a cone is called the apex. The height of a cone is the distance from the center of the base to the apex. Some cone-shaped objects are traffic cones and ice cream cones.

Cylinder



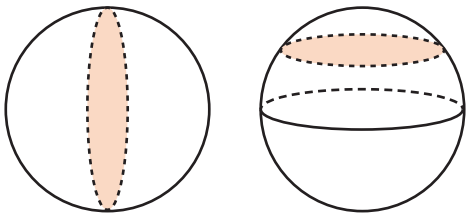
A ***cylinder*** is a solid with two circular bases that are congruent and parallel. An example of a cylinder is a metal can or a plumbing pipe.

Cross Sections

A cross section is the shape formed by cutting straight through a solid. The same solid can have cross sections of different shapes. A cross section that is parallel to the base of a polyhedron will be the same shape as the base. Look at the cross sections of various geometric solids below. The shape of each cross section is listed below the solid.

Cross Sections of a Sphere

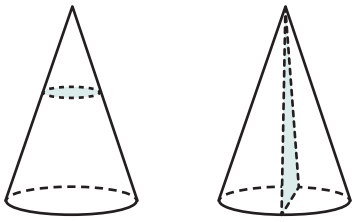
Every cross section of a sphere is a circle.



circle circle

Cross Sections of a Cone

The horizontal cross section of a cone is a circle. The vertical cross section through the apex is a triangle.

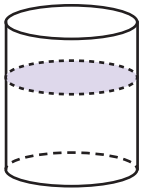


circle triangle

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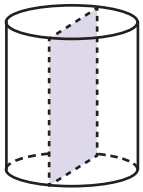
Cross Sections of a Cylinder

Cross sections parallel to the bases are circles.



circle

Cross sections perpendicular to the bases are rectangles.

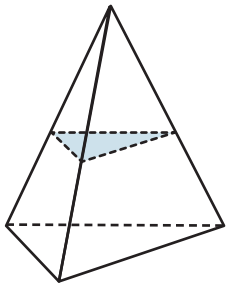


rectangle

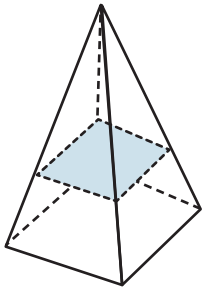
Cross Sections of Pyramids

Below are just a few examples of possible vertical and horizontal cross sections for a triangular pyramid, square pyramid, and hexagonal pyramid.

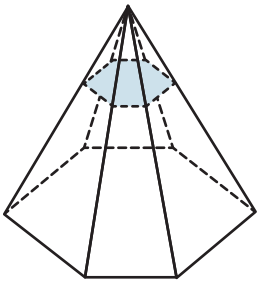
Horizontal cross sections (parallel to the base):



triangle

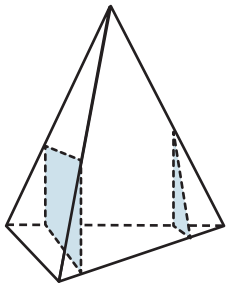


square

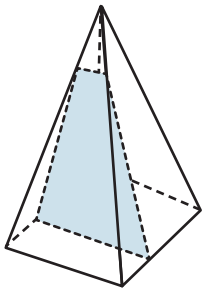


hexagon

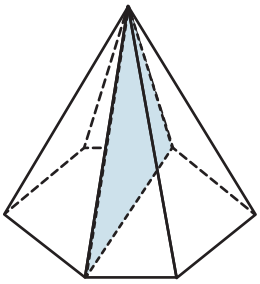
Vertical cross sections (perpendicular to the base):



trapezoid or triangle



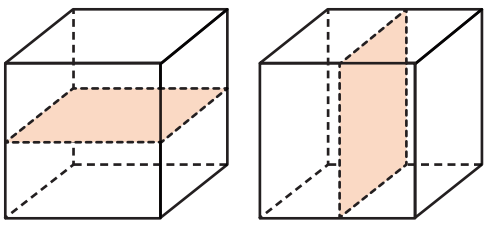
trapezoid



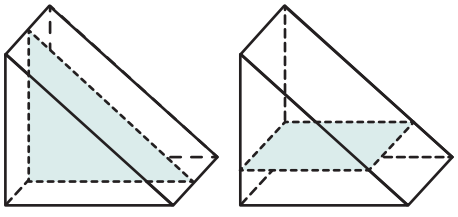
triangle

Cross Sections of Prisms

In a cube, like the one below, cross sections parallel to the bases and cross sections perpendicular to the bases are squares.



In a triangular prism, cross sections parallel to the bases are triangles. Cross sections perpendicular to the bases are rectangles.



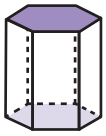
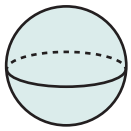
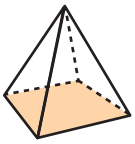
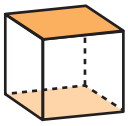
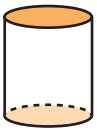
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PRACTICE

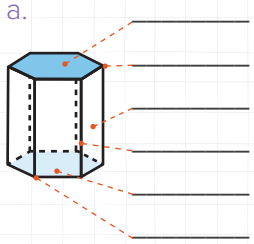
1. Fill in the table with the general name for each geometric solid. Choose from the word bank below.

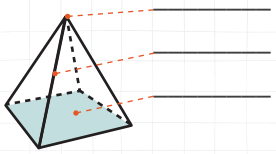
Word Bank:

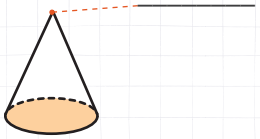
sphere	prism	cone
pyramid	cylinder	

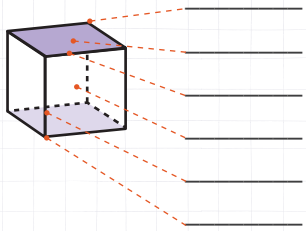
Solid	Name
	
	
	
	
	
	
	
	
	

2. For each shape, write whether the indicated part is a *face*, *edge*, *vertex*, or *apex*.

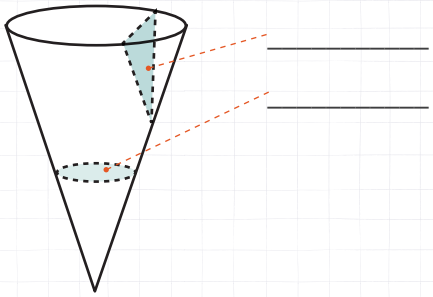
a. 

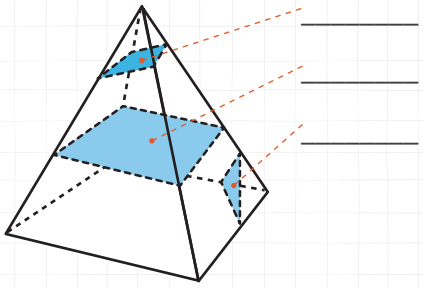
b. 

c. 

d. 

3. Identify the shape of the indicated cross sections in each solid.

a. 

b. 

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4. For each geometric solid listed below, draw an example of the solid below the name. Then find its name in the word search.

Rectangular Prism

Cylinder

Cube

Square Pyramid

Sphere

Triangular Pyramid

Square Prism

Cone

Pentagonal Prism

For fun, here are some additional words to find in the word search:

VERTEX
EDGE
FACE
APEX

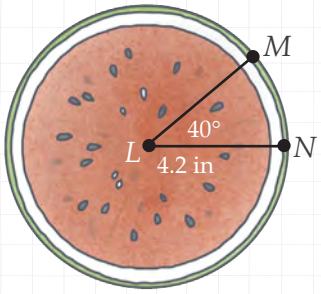
P	T	R	I	A	N	G	U	L	A	R	P	Y	R	A	M	I	D	A	U
B	E	T	F	D	D	J	D	K	Z	V	R	E	F	O	V	M	Q	P	Q
H	Z	N	C	P	M	A	V	D	B	E	F	Q	E	D	U	X	L	E	C
U	M	N	T	O	G	N	W	X	P	G	V	M	N	C	A	E	L	X	Y
W	M	J	R	A	N	S	O	Y	M	D	L	L	W	F	G	X	O	I	B
T	B	S	L	E	G	E	S	C	L	U	Y	E	F	T	Q	B	O	T	C
V	C	V	U	I	C	O	T	T	Z	O	K	D	P	U	T	S	L	A	Y
Z	E	U	X	E	R	T	N	M	G	K	D	G	B	O	I	F	N	B	L
M	X	R	I	H	F	L	A	A	S	L	W	E	U	L	U	D	C	E	I
E	W	I	T	X	F	S	S	N	L	W	L	Q	D	V	I	V	S	Y	N
Q	U	K	Z	E	Y	T	Q	S	G	P	Q	A	U	E	M	S	T	E	D
O	F	R	B	W	X	O	G	U	B	U	R	S	P	H	E	R	E	F	E
H	J	I	Y	S	D	R	R	I	A	X	L	I	V	U	J	Y	T	C	R
K	B	C	C	K	P	Y	I	V	L	R	I	A	S	Q	G	T	R	G	P
S	M	I	U	D	A	D	D	Z	R	F	E	X	R	M	Z	L	Q	X	Y
A	P	S	B	B	J	J	P	Y	M	W	A	P	P	P	M	I	H	B	G
Z	A	M	B	R	E	P	Z	T	I	V	Z	C	R	D	R	L	N	M	P
D	N	Z	D	I	M	R	H	B	P	M	U	S	E	I	U	I	X	I	X
R	L	R	N	A	S	V	B	P	J	C	X	R	G	R	S	Z	S	R	Z
M	L	S	Q	U	A	R	E	P	Y	R	A	M	I	D	M	M	M	M	L



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REVIEW

1. Use the image below to answer the questions. When necessary, round to the nearest tenth.



a. What is the area of the watermelon slice (entire circle)? L84

$A \approx$ _____

b. If the watermelon slice is cut into wedges the size of sector *MLN*, how many wedges can be cut from the slice? L92

_____ wedges

c. The outside edge of the watermelon slice is called the rind. What is the length of the rind on the wedge represented by sector *MLN*? L92

✦ Hint: Find the length of $\overset{\frown}{MN}$.

_____

2. Lewis borrowed $3,000 from his aunt to pay for graduate school. Four years after borrowing the money, he wants to pay it back with 2.5% simple interest. How much interest will he pay? L49

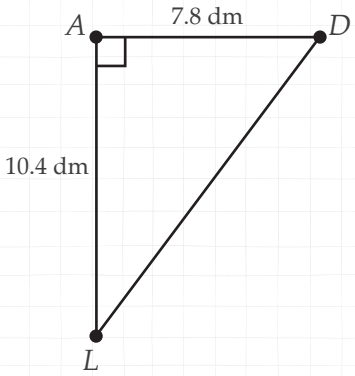
✦ Hint: The simple interest formula is $I = Prt$.

_____

3. What is the length of *LD* in the figure below? L81

✦ Hint: Use the Pythagorean Theorem.

_____



4. After helping her dad build a shed, Janet found three leftover pieces of wood that were 6 in, 9 in, and 1 ft long. Can a triangle be formed from the pieces of wood? L71

✦ Hint: First convert all dimensions to the same units. To form a triangle, the sum of any two sides must be greater than the third side.

_____



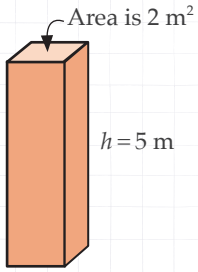
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PRACTICE

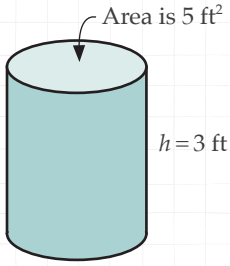
1. Use the height and the given area of the base to find the volume of each solid.

✦ Hint: See the Key Information box in the Lesson Overview.

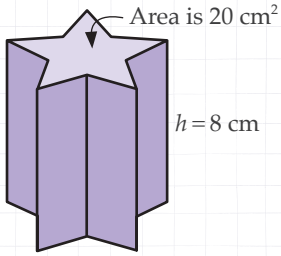
a.



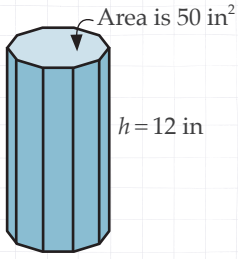
b.



c.

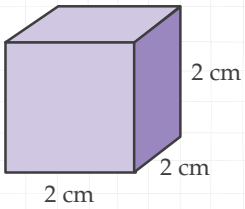


d.

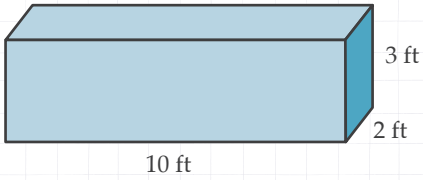


2. Find the volume of each solid below. Round to the nearest hundredth, if needed.

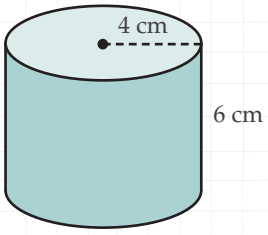
a.



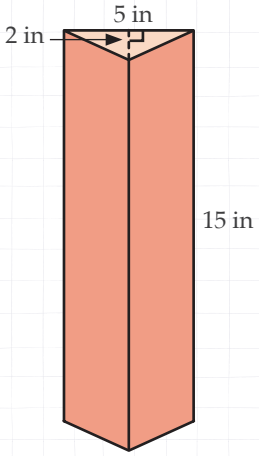
b.



c.



d.

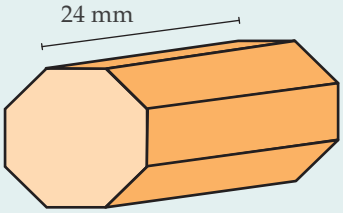


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Volume of Other Geometric Solids

WARM-UP

Find the volume of the octagonal prism below if the area of the base of the prism is 50 mm^2. Remember, the volume of a prism is found by multiplying the area of the base by the height.



Volume: _____

LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Notes section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

App Video

VIDEO NOTES

Traffic Cone

Area of the circle:

$A = \pi(____)^2$

$A = _____\ \pi$

Area: _____ in^2

Volume of the cone:

$V = \frac{1}{3}(____)(____)$

Volume: _____ in^3

Metronome

Area of the triangle:

$A = \frac{1}{2}(____)(____)$

Area: _____ cm^2

Volume of the pyramid:

$V = \frac{1}{3}(____)(____)$

Volume: _____ cm^3

Cannonball

Volume of the sphere:

$V = \frac{4}{3}\pi(____)^3$

$V = \frac{4}{3}\pi(____)$

$V = ____\ \pi$

Volume: _____ in^3

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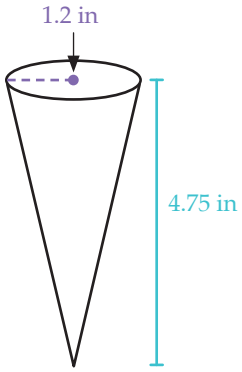
LESSON OVERVIEW

Volume is used in things like cooking, baking, medicine, and engineering. Every three-dimensional shape has volume. The volume of different solids can be found using formulas. Volume is expressed in cubic units.

Volume of a Cone

The volume of a cone is found using the formula $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height of the cone. The height of a cone is the distance from the apex to the base.

Example 1: An office waiting room has a water tank with small cone-shaped cups. Below is an image with measurements. Find the volume of the cup to the nearest hundredth.



To use the volume formula, calculate the area of the base first.

Area of Base (B):

The base of a cone is a circle.

$A = \pi r^2$

$A = \pi(1.2)^2$

$A = 1.44\pi$

$A \approx 4.52$

The area of the circle is approximately 4.52 in^2. This is the value of B in the volume formula.

Volume of Cone:

Use 4.52 for B.

$V = \frac{1}{3}Bh$

$V = \frac{1}{3}(4.52)(4.75)$

$V \approx 7.16$

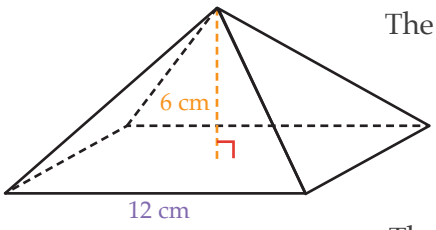
The volume of the cup is about 7.16 in^3.

Fun Fact: This is almost 4 fl oz.

Volume of a Pyramid

The volume of a pyramid is found using the formula $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height of the pyramid. The height of a pyramid is the distance from the apex to the base.

Example 2: Find the volume of the square pyramid.



Area of Base (B):

The base of this pyramid is a square.

$A = s^2$

$A = 12^2$

$A = 144$

The area of the square is 144 cm^2. This is the value of B in the volume formula.

Volume of Pyramid:

Use 144 for B.

$V = \frac{1}{3}Bh$

$V = \frac{1}{3}(144)(6)$

$V = 288$

The volume of the pyramid is 288 cm^3.

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Example 3: The Great Pyramid of Giza is the biggest Egyptian pyramid. Find the volume of the pyramid using the original height of about 481 feet and the original base side length of about 756 feet. (The base is square.)

To use the volume formula, calculate the area of the base first.

Area of Base (B):

The base of this pyramid is a square.

$A = s^2$

$A = 756^2$

$A = 571536$

The area of the square is 571,536 ft^2. This is the value of B.

Volume of Pyramid:

Use 571,536 for B.

$V = \frac{1}{3}Bh$

$V = \frac{1}{3}(571536)(481)$

$V \approx 91636272$

The volume of the pyramid is about 91,636,272 ft^3.

Volume of a Sphere

The volume of a sphere is found using the formula $V = \frac{4}{3}\pi r^3$.

Example 4: A spherical glass vase is filled with water beads. Find the volume of the sphere if the radius of the vase is 6 inches.



$V = \frac{4}{3}\pi r^3$ Substitute 6 for the radius.

$V = \frac{4}{3}\pi(6)^3$ First, find 6 cubed.

$V = \frac{4}{3}\pi(216)$

$V = 288\pi$

$V \approx 904.78$

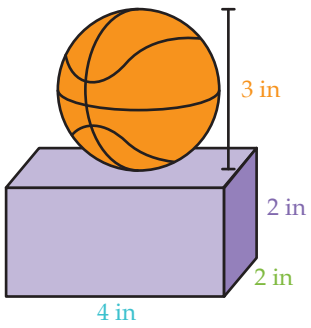
Fun Fact: This is almost 4 gallons.

The volume of the vase is approximately 904.78 in^3.

Volume of Composite Solids

The volume of composite solids can be found by finding the volume of each three-dimensional figure that makes up the composite solid and adding the volumes together.

Example 5: Calvin has a coin bank that is a square prism base with a basketball on top. Given the measurements below, find the volume of the coin bank.



Volume of Square Prism:

$V = lwh$

$V = 4 \cdot 2 \cdot 2$

$V = 16$

Volume of Sphere:

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(1.5)^3$ Use 1.5 for r since the diameter is 3.

$V = \frac{4}{3}\pi(3.375)$

$V = 4.5\pi$

$V \approx 14.14$

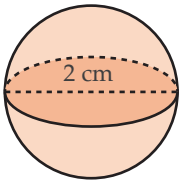
Add the two volumes together to find the volume of the composite solid.

$16 + 14.14 = 30.14$ The volume of the coin bank is approximately 30.14 in^3.

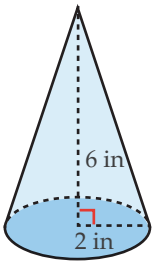
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3. Begin at START. Find the volume of the shapes shown to work your way through the maze. Round each volume to the nearest whole number.

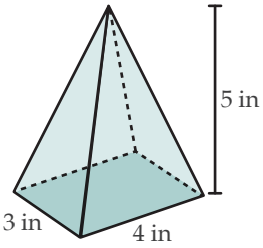
START



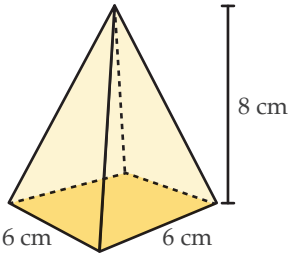
4 cm^3



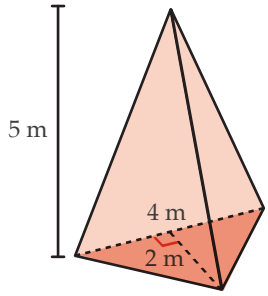
4 in^3



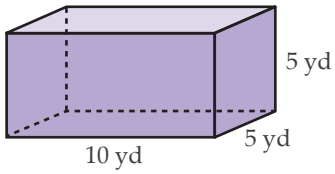
96 m^3 | 34 cm^3 | 25 in^3 | 7 m^3 | 20 in^3



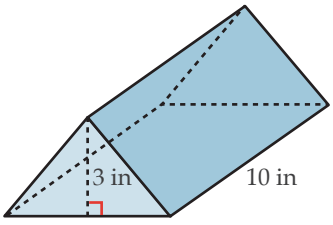
6 m^3



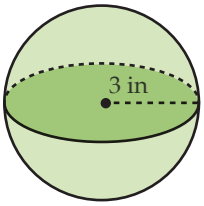
40 m^3



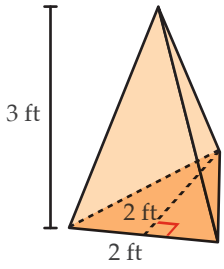
16 m^3 | 288 m^3 | 40 m^3 | 250 yd^3 | 20 yd^3



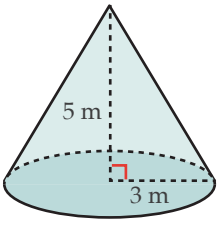
38 in^3



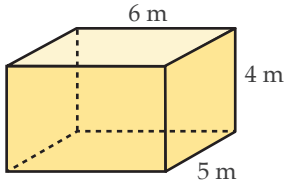
6 ft^3



150 m^3 | 113 in^3 | 60 m^3 | 14 in^3 | 3 ft^3



47 m^3



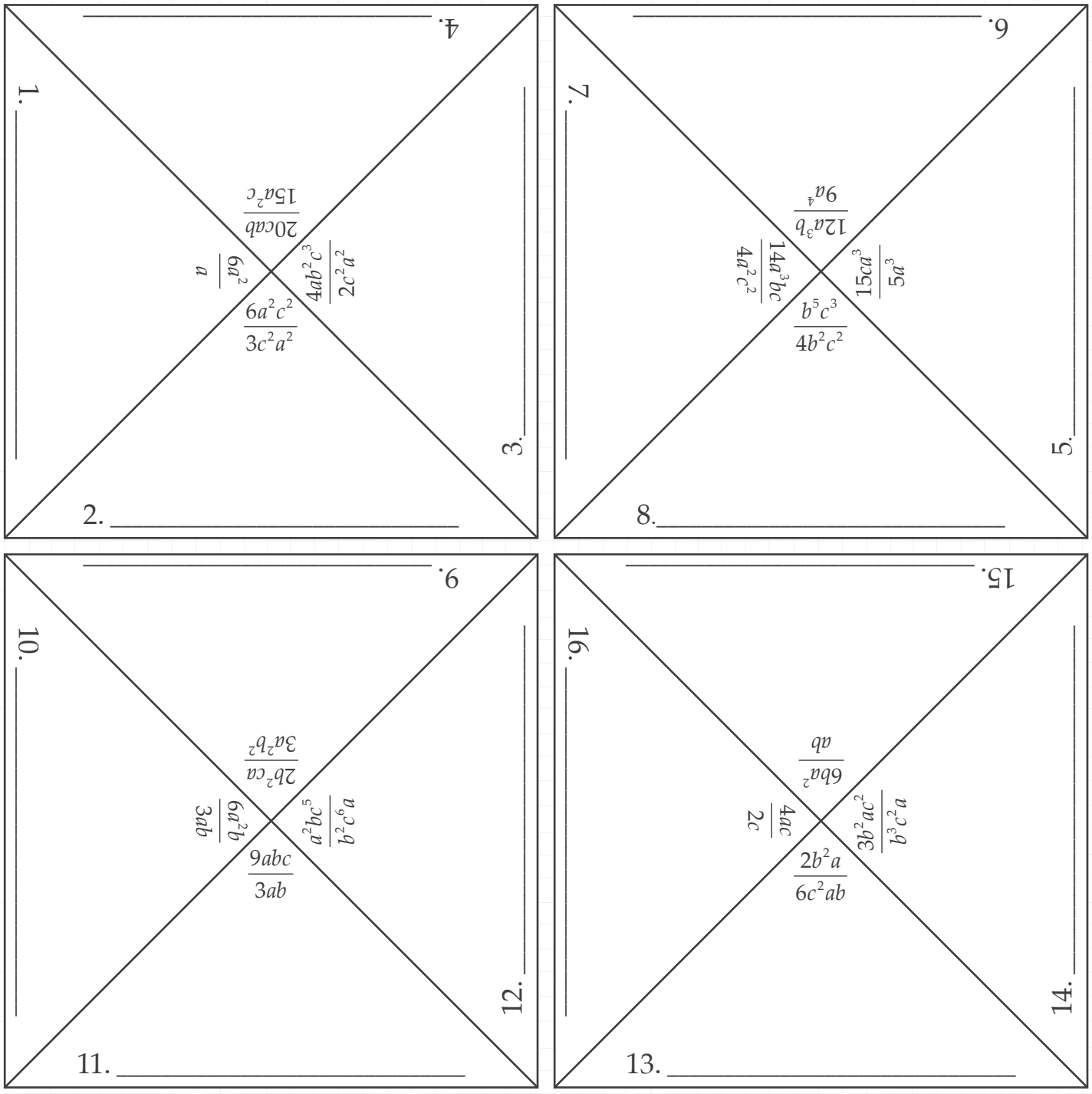
120 m^3

FINISH

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PRACTICE

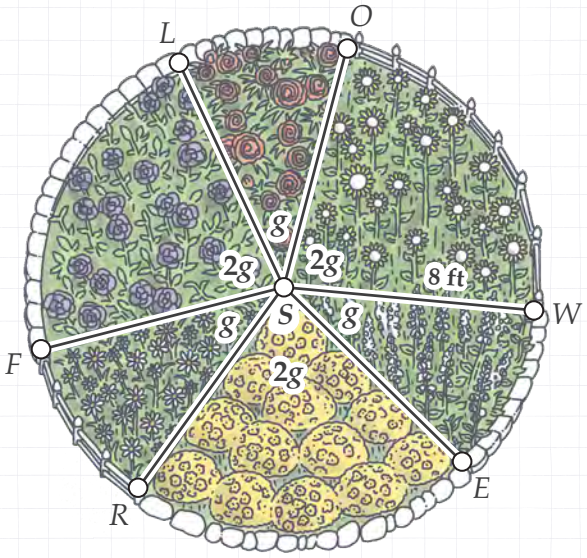
Cut out the squares below. Simplify the four rational expressions on each square and write the simplified expression on the line provided. Once all expressions have been simplified, match the sides of the squares together if they simplify to the same expression. Each square will match to two other squares.



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REVIEW

1. Richard is building a circular flower garden according to the design shown below.



a. Write and solve an equation to find the value of g. Then find the measure of each angle. L80

$g =$ _____ $m\angle FSL =$ _____

$m\angle LSO =$ _____

b. Find the area and circumference of the garden rounded to the nearest hundredth. L84

Area: _____

Circumference: _____

c. Richard will plant peonies in sector LSO and carnations in sector FSL. What is the area of each of these sectors? L92

LSO: _____ FSL: _____

d. Richard is considering placing a different type of border (such as boulders, concrete pavers, wooden pickets, etc.) around each sector of his garden. To calculate how much of each border he will need, Richard must find the arc length of each section. Find the lengths of $\overset{\frown}{FL}$ and $\overset{\frown}{LO}$ to the nearest hundredth. L92

$\overset{\frown}{FL}$: _____ $\overset{\frown}{LO}$: _____

e. Fabulous Flowers, the company from which Richard will buy seeds, completes a quality check on a randomly chosen seed packet from the first 100 packets filled each day. Then they quality check every 50th packet after the randomly chosen packet. L103

Is this sampling method biased or unbiased?

_____

Fill in the blanks: This is an example of a

_____ sample.

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Logic Lesson 4

Vernal Pool Ecosystem

Vernal pools are seasonal wetlands that typically fill with water from melting snow or rain showers in spring and dry up by late summer or early fall. Certain species of animals and plants rely on these temporary pools for survival. In this lesson, you'll learn more about vernal pool ecosystems as you complete several number and logic puzzles.

There is no video or review for this lesson.

Studious Students

Vernal pools vary greatly in size. Some are as small as an ordinary rain puddle. Others resemble a shallow lake, such as California's nearly 100-acre vernal pool named Olcott Lake.

Suppose a group of students from a science club surrounded a small circular vernal pool in order to make observations. The students were evenly spaced, and the seventh student was standing directly opposite the 18th student. How many students were in the group?

✦ Hint: Draw a picture.

_____ students

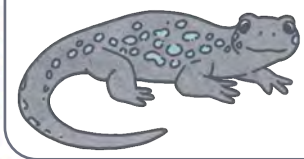
Salamander Surveys

Some amphibians, such as salamanders and frogs, lay eggs in vernal pools. The eggs hatch as the pools fill, and the larvae or tadpoles develop and metamorphose as the water begins to evaporate. By the time the pools dry again, the animals have grown into adults that hop or crawl to new homes in the surrounding land.

Suppose people living near some vernal pools were surveyed regarding which salamander species they had recently seen in the area. If exactly 91.2% of people returned the survey, what is the fewest number of people that could have been surveyed?

✦ Hint: Rewrite 91.2% as a simplified fraction.

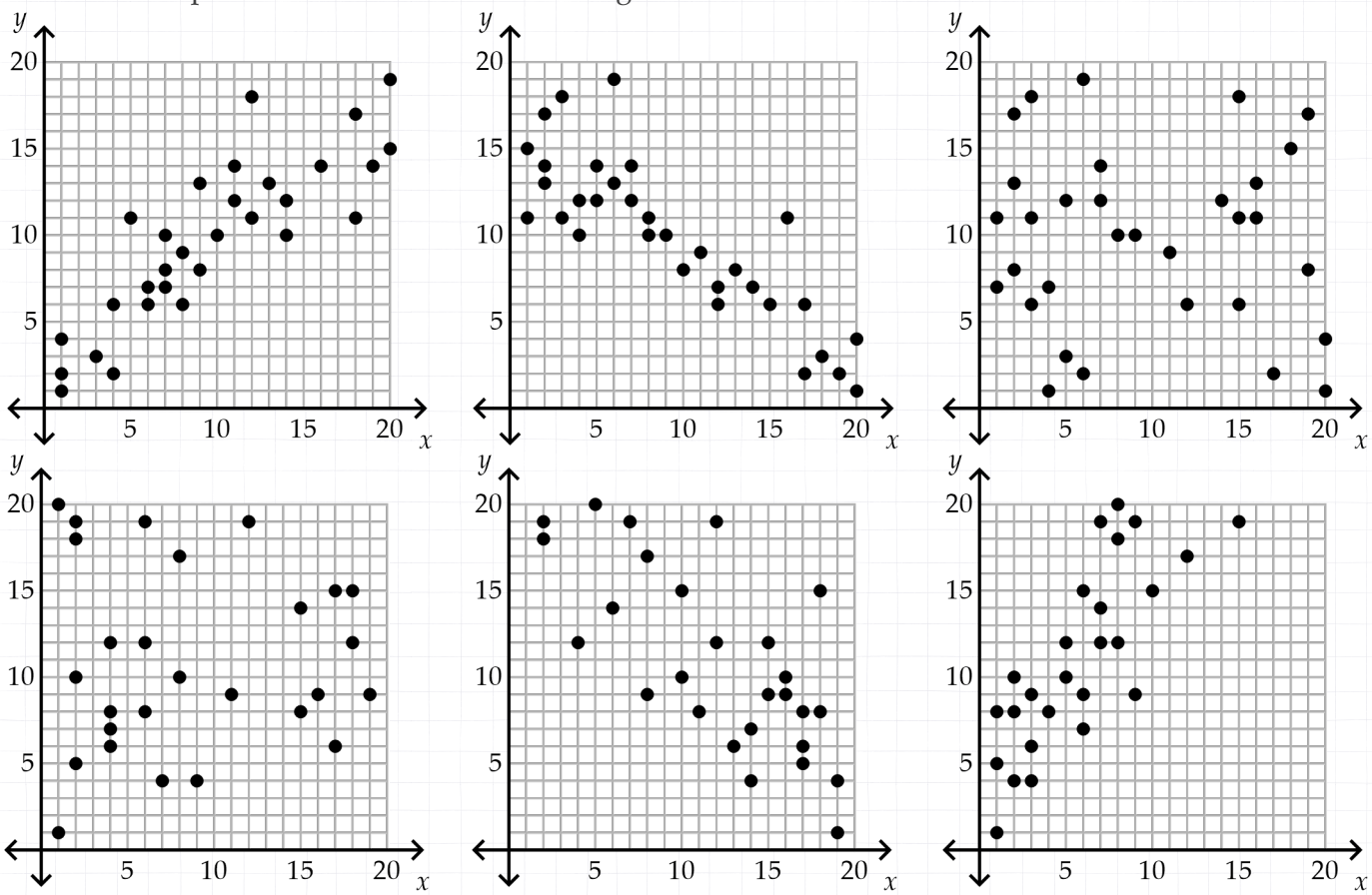
_____ people



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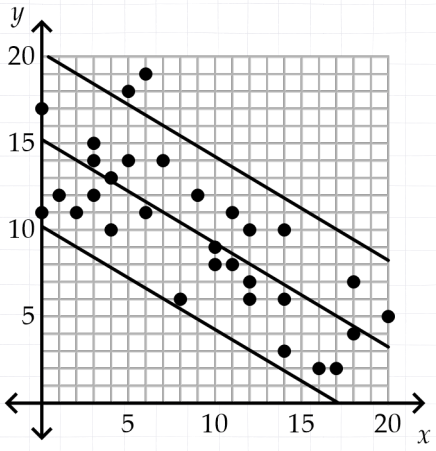
PRACTICE

Use the scatter plots below for Problems 1 through 4.

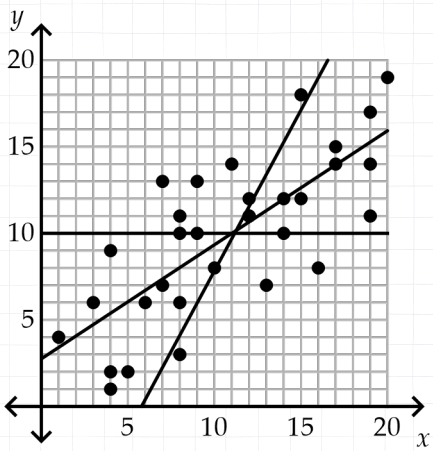


1. Circle any scatter plots that appear to show a positive correlation in yellow.
2. Circle any scatter plots that appear to show a negative correlation in blue.
3. Circle any scatter plots that do not appear to show a correlation in red.
4. For the scatter plots that appear to show a correlation, draw an estimated line of best fit on each graph.
5. Each of the scatter plots below shows three lines. Circle the line that best fits the data.

a.



b.



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2. Use the word bank to fill in the blank(s) in each sentence. Then find the missing words in the word search.

a. The _____ is the middle number of a data set.

b. The _____ of a data set is obtained by adding all the data values and dividing by the number of data values.

c. A graph of data will look _____ if the mean and median are close to each other.

d. When the mean is _____ than the median, a graph will be right _____.

e. If the data has some outliers that are significantly less than the majority of other data values, the graph will be _____ skewed, and the mean will be _____ than the median.

f. A box plot of right-skewed data will have a _____ whisker on the right.

g. The most common data value in a set is the _____.

h. A data set with one mode is called _____.

i. A _____ data set will have a graph with two peaks.

WORD BANK

bimodal
larger
left
less
longer
mean
median
mode
skewed
symmetric
unimodal

C	M	K	W	C	K	K	O	E	C	C	C	C	I	L	J	A	V	E	U
M	M	R	I	O	W	V	M	G	L	S	A	J	H	S	S	N	E	P	H
B	E	F	P	T	H	P	K	H	V	L	N	K	Q	F	H	R	H	Y	Z
O	Y	V	V	S	R	X	G	X	A	Z	M	T	O	Z	S	T	F	J	W
D	E	V	W	L	Y	O	U	T	J	C	O	L	U	L	N	N	O	N	I
D	G	P	K	V	Z	Z	T	U	L	Q	M	K	O	Y	Q	V	M	U	E
U	C	R	U	K	A	Z	F	W	L	I	E	N	W	N	U	U	E	N	K
N	S	X	S	L	C	Y	V	V	S	D	D	T	G	L	G	I	A	I	D
V	H	K	Y	J	E	S	B	X	Q	N	I	L	J	Z	Q	E	N	M	N
U	W	S	X	K	J	S	O	B	E	I	A	C	T	N	R	F	R	O	R
E	D	D	H	D	R	R	S	N	I	U	N	F	S	U	P	L	Q	D	N
F	F	O	G	E	O	K	B	M	M	G	L	W	T	I	F	W	Y	A	S
S	Y	Q	B	W	Z	Q	U	S	Y	M	M	E	T	R	I	C	L	L	A
F	N	C	J	I	M	O	D	E	X	R	E	O	G	D	B	V	A	Q	K
Y	V	L	K	G	M	E	O	V	U	Y	R	N	O	T	D	C	R	M	I
F	M	B	P	I	F	O	S	K	E	W	E	D	B	Z	I	X	G	P	R
P	L	F	X	I	O	J	D	P	W	T	D	J	P	I	D	L	E	F	T
L	E	W	O	J	H	V	O	A	N	V	C	H	V	J	V	I	R	E	X
L	A	P	A	Q	I	N	Q	G	L	F	F	H	I	K	H	M	W	J	C
A	Q	N	T	N	D	B	M	K	I	J	C	G	H	H	C	G	X	B	X

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REVIEW

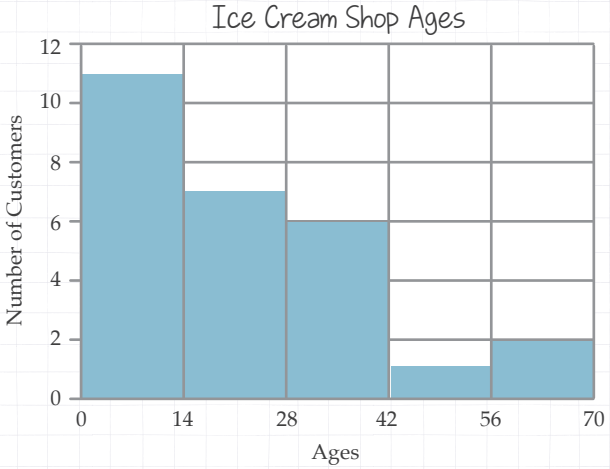
1. The data set below represents the resting heart rate of six high school cross-country runners. Find the mean, median, mode, and range of the data set. L106

Data set: 53, 42, 60, 48, 51, 64

a. Mean: _____ b. Median: _____

c. Mode: _____ d. Range: _____

2. The histogram below represents the ages of customers eating ice cream cones at Ina's Ice Cream Shop.



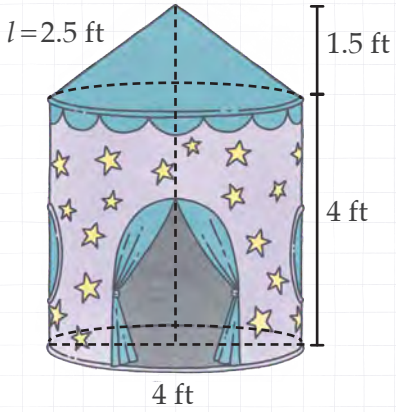
a. Is the histogram *left skewed*, *right skewed*, or *symmetric*? L110

_____

b. Based on your answer to Part A, compare the median and mean of the data using <, >, or ≈. L110

mean ◯ median

3. Niya is making a play tent as a gift for her niece's birthday. The design is shown below. The diameter of the tent base is 4 ft. The height of the tent in the center is 5.5 ft, and the height of the cylindrical portion is 4 ft. The height of the cone roof is 1.5 ft. The tent will include a fabric floor.



a. To figure out how much fabric she will need to make the tent, Niya needs to find the surface area of the composite solid. What is the surface area of the tent? Round to the nearest tenth. L96

✦ Hint: The cylinder part of the tent has just one circular base, and the cone portion does not include a base. See Reference Chart for formulas.

_____

b. Fabric is typically sold by the square yard. Convert the surface area to square yards. Round to the nearest whole number. L55

_____

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Sample Space

WARM-UP

Find the probabilities below for one roll of a six-sided die. Write each probability as a fraction in simplest form.

a. What is the probability of rolling a 3? _____

b. What is the probability of rolling a number less than 3? _____

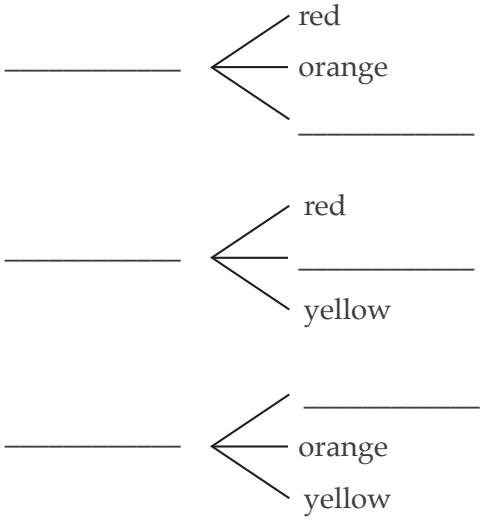
c. What is the probability of rolling an even number? _____

LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Notes section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

App Video

VIDEO NOTES



Outcomes:

RR	RO	____
OR	____	OY
____	YO	YY

Probability: $\frac{\square}{9}$ or _____ %

1, ____ 2, ____ 3, ____ 4, ____

1, ____ 2, ____ 3, ____ 4, ____

1, ____ 2, ____ 3, ____ 4, ____

1, ____ 2, ____ 3, ____ 4, ____

Probability: $\frac{2}{\square} = \frac{\square}{\square}$ or _____ %

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LESSON OVERVIEW

When finding the likelihood of more than one event occurring, it can help to see all the possible outcomes visually. A ***sample space*** is the set of all possible outcomes of a probability experiment. There are many ways to represent the sample space, including lists, tree diagrams, and tables. Representing the sample space is an important part of understanding and calculating probabilities.

Organized Lists

Writing the sample space in a list makes it easier to count the desired outcomes and total outcomes.

Example 1: Suppose two tiles are randomly drawn from a bag containing the five tiles shown below. Find the probability of drawing the triangle tile and circle tile.



Make a list of the possible combinations when drawing two of the five tiles. Start by pairing one shape with all other shapes. Then pair another shape with the three shapes it has not been paired with, and so on, until all possible pairings are listed. This helps to ensure no possible outcome is missed. The sample space is represented below with a list.

circle - star	star - triangle	triangle - pentagon	pentagon - heart
circle - triangle	star - pentagon	triangle - heart	
circle - pentagon	star - heart		
circle - heart			

There are 10 possible outcomes when drawing two of the five tiles. Drawing the triangle and the circle is one of these 10 outcomes (highlighted above).

The probability of drawing the triangle and circle is $\frac{1}{10}$, or 10%.

Tree Diagrams

A tree diagram can be used to find all possible outcomes for a repeated event, like flipping a coin. In a tree diagram, the outcomes of each event are in separate columns.

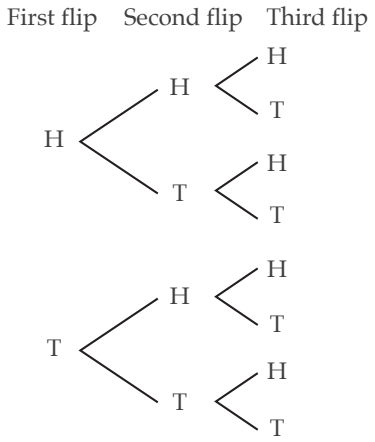
Example 2: Find the probability of flipping a coin three times and getting heads at least twice.

A tree diagram is shown to the right. A list of the possible outcomes can be made from a tree diagram. Start at the top and work down through all the options for each flip.

HHH	HHT	HTH	HTT
THH	THT	TTH	TTT

There are 8 possible outcomes. There are 4 outcomes for flipping at least two heads (highlighted above).

The probability of flipping at least two heads is $\frac{4}{8} = \frac{1}{2}$, or 50%.



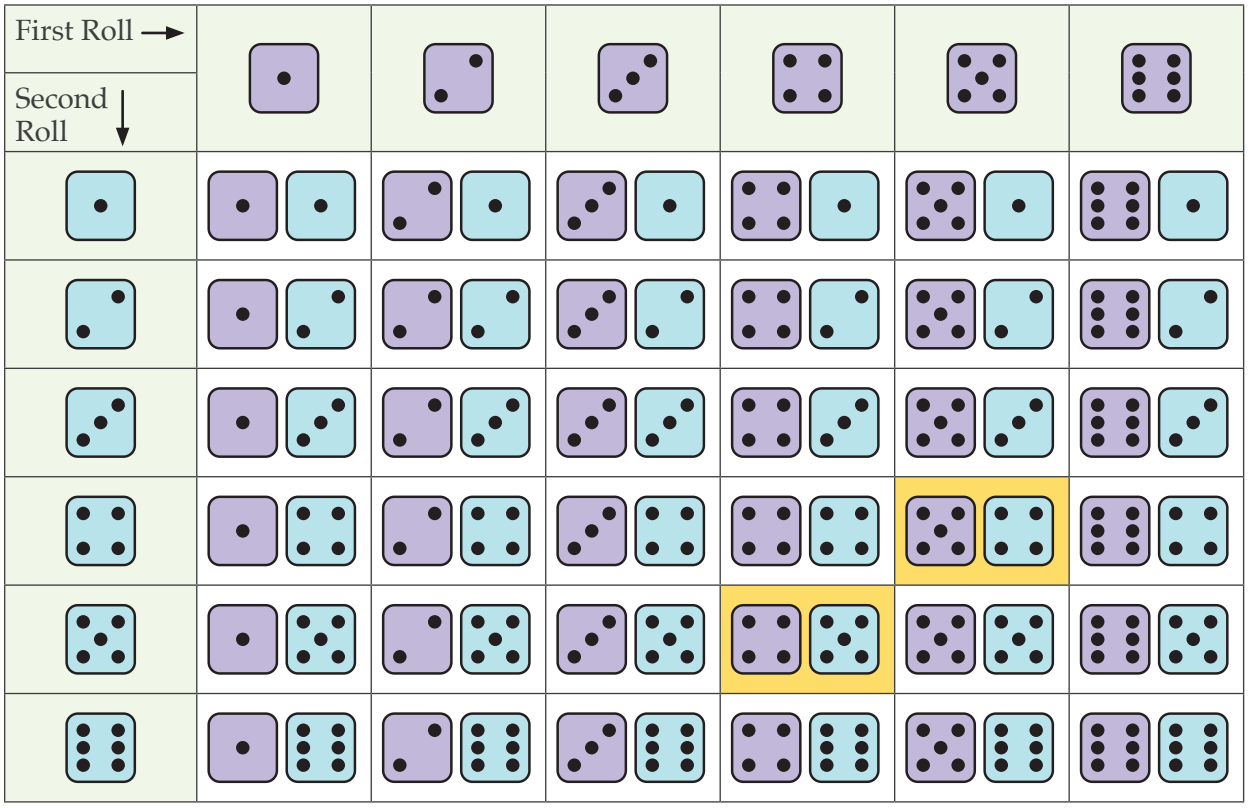
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Tables

A table is especially useful for probabilities with several possible outcomes.

Example 3: Find the probability of rolling a 5 and a 4 when rolling a pair of dice.

The table below shows the possible outcomes of rolling two dice. There are 36 possible outcomes.



There are two outcomes for rolling a 5 and a 4 (highlighted above). The purple die could land on 5 and the blue die could land on 4, or the purple die could land on 4 and the blue die could land on 5.

The probability of rolling a 5 and 4 is $\frac{2}{36} = \frac{1}{18}$, or $5.\overline{5}\%$.



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6. a. Are the experimental probabilities after 10 rolls equal to the theoretical probabilities? _____

   b. Are the experimental probabilities after 40 rolls equal to the theoretical probabilities? _____

   c. Why does experimental probability sometimes differ from theoretical probability?

   _____

   _____

7. Add the frequencies found in Problems 2 and 3 together and record the total frequencies in the table below. This table represents rolling the die 50 times.

Roll	Frequency
1	
2	
3	
4	
5	
6	

Find the experimental probability of rolling each number out of 50 rolls.

	Fraction:	Percent:
1.	_____	_____
2.	_____	_____
3.	_____	_____
4.	_____	_____
5.	_____	_____
6.	_____	_____

8. Fill in the table representing the sample space of rolling two dice. Some examples are given.

First Roll → / Second Roll ↓						
	1, 1					
					5, 2	
						
		2, 4				
						
						

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Unit 4 Review

Complete this Unit Review to prepare for the Course Assessment. There is no video, lesson, or practice. Because Unit Reviews include practice for an entire unit, they may take longer than regular lessons, and students may decide to take two days to finish.

Round all answers to the nearest hundredth, unless otherwise specified.

Posey, Toby, and Winston held a summer Bible camp for the younger children in their neighborhood.

Scale Factor with Area
Lesson 91

1. Winston and Toby decided to pass flyers out to all their neighbors. They created a 4-in by 6-in design and then enlarged each dimension by a scale factor of 2.

   a. What was the area of the initial design?

   _____

   b. What are the dimensions of the flyer?

   _____ and _____

   c. What is the area of the flyer?

   _____

   d. By what factor did the area of the design increase?

   _____



Polynomials
Lessons 99–102

2. With a lot of facial expressions, Posey read Bible stories that the children enjoyed. Simplify the mathematical expressions below.

   a. $x^2y + 3x - 2x^2y - 1 - x$

   _____

   b. $ab^2(b - 2a)$

   _____

   c. $\frac{4n^2m}{6nm^2}$

   _____



3. Use the GCF to factor the expression below.

   $12x^2y + 8xz$

   _____

Population and Sampling Methods
Lesson 103

4. The week before camp, Toby surveyed neighboring kids about their favorite snacks. He asked the first 15 kids he saw one day at an ice cream shop.

   a. Is this sample random? _____

   b. Is this sample biased? _____



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Measures of Central Tendency
Lessons 106, 107

9. As part of an activity about Jesus providing fish for fishermen (Luke 5), some of the children at Bible camp were given fish-shaped crackers as a snack. Older kids received bigger piles of crackers. Ten children counted their crackers. The number of crackers in each pile is shown below.

35, 47, 39, 52, 62, 46, 52, 63, 42, 39

a. What is the mode number of crackers?

✦ Hint: There is more than one mode.

Mode: _____

b. What is the median number of crackers?

✦ Hint: First, put the data in numerical order.

Median: _____

c. One child chose not to count his crackers. If the average number of crackers among the 11 children was 48, how many crackers did this last child have?

_____



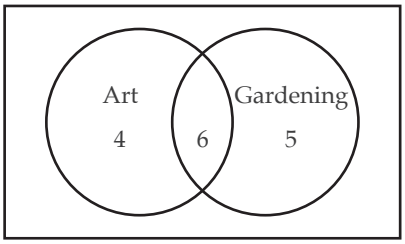
Probability
Lessons 111–114

10. 7 of the 15 children at camp were boys, and 8 of the 15 children were under age six. If a random child is chosen, what is the probability (as a fraction) that the child will be age six or older?

_____



11. The Venn diagram shows how many children participated in activities. What is the probability that a child did just one of the activities, but not both?

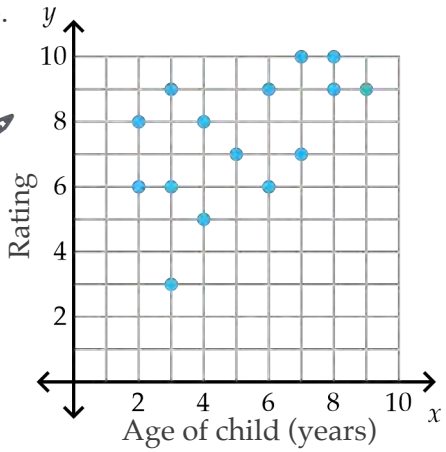


_____

Scatter Plots
Lesson 109

12. After the camp, Posey gave out surveys to the families. She created a scatter plot to see if there was a relationship between a child's age and how well he or she rated the camp.





a. What type of correlation is shown on the scatter plot? Circle one.

positive / negative / no correlation

b. Draw a line of best fit on the scatter plot.

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Course Review

- This lesson is a multiple-choice Course Review to prepare for the Course Assessment. There is no video, lesson, or practice.
- This review may take longer than regular lessons, and students may decide to take two days to finish.
- Lesson numbers are given for each question so students can review lessons as needed.

Circle the letter of the correct answer for each problem. Simplify all answers and round to the nearest hundredth when necessary.

1. Convert 2.6 to a mixed number. L5

   a. $\frac{13}{5}$ b. $2\frac{6}{10}$ c. $2\frac{3}{5}$ d. $2\frac{1}{6}$

2. The decimal equivalent of $3\frac{2}{7}$ is L4

   a. terminating. b. repeating.

3. The prime factorization of 132 is L2

   a. $2 \cdot 3 \cdot 11$. b. $4 \cdot 3 \cdot 11$.

   c. $11 \cdot 12$. d. $2 \cdot 2 \cdot 3 \cdot 11$.

4. Evaluate $3^2\left(1-2\cdot(-4)\right)$. L21

   a. 81 b. –63 c. 54 d. –42

5. Evaluate $\dfrac{\frac{3}{4}-\frac{1}{2}}{2}$. L22

   a. $\frac{1}{2}$ b. $\frac{1}{8}$ c. 1 d. $\frac{1}{4}$

6. Evaluate the expression $3a+b^2$ when $a=2$ and $b=5$. L24

   a. 16 b. 31 c. 19 d. 5

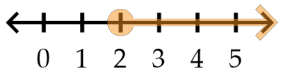
7. Solve $2t-1=1.8$ for t. L33

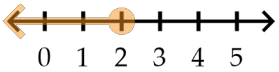
   a. 5.6 b. 0.9 c. 0.4 d. 1.4

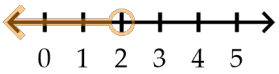
8. Solve $-\frac{2}{3}(p+1)=4$ for p. L35

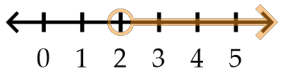
   a. –7 b. –5 c. 7 d. 5

9. Graph the solutions to $4x \geq x+6$. L39

   a. 

   b. 

   c. 

   d. 

10. A fruit basket contains three oranges for every two apples. The basket contains 30 pieces of fruit. How many oranges does it contain? L42

    a. 10 b. 18 c. 12 d. 15

11. Which of the following proportions shows the relationship 6 is to 4 as x is to 6? L41

    a. $\frac{6}{4}=\frac{x}{6}$ b. $6 \cdot 4 = x \cdot 6$ c. $\frac{6}{6}=\frac{4}{x}$

12. Convert $\frac{3}{8}$ to a percent. L5, L46

    a. 0.125% b. 0.375%

    c. 12.5% d. 37.5%

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Course Assessment

- This assessment covers concepts taught in Math 7. Problems are designed to assess multiple skills. Read the instructions carefully and do not rush through the problems.
- Students may use the Reference Chart for the assessment. Lesson numbers are given by each problem so you can review lessons for any answers that are incorrect.
- Calculators may be used where needed on the entire assessment.



1. Simplify the fraction using prime factorizations. L2, L3

$\frac{224}{672}$

_____

2. Convert the fractions to decimals. L5

a. $\frac{1}{8}$ _____ b. $\frac{3}{200}$ _____

3. Convert the decimals to fractions. L5

a. 0.145 b. 0.08

_____ _____

4. Evaluate the exponential expressions. L13, L14

a. 5^3 b. $\left(\frac{1}{2}\right)^3$ c. 3^{-4}

_____ _____ _____

5. Evaluate the expressions. L21, L22

a. $\left(5+11^2\right)\div 3$ _____

b. $6 \bullet 4 - 19 + 2^3$ _____

6. Simplify the expressions. L23

a. $17z - 4z + 16 + 13$ _____

b. $4t^2 + 3s - 7t^2 + 5s$ _____

7. Evaluate the expressions when $a = 3$ and $b = -5$. L24

a. $\frac{10a}{3b}$ b. $3a - 4b$

_____ _____

8. Tanner and his brother Tony raise dairy cows. Let m represent the number of gallons of milk Tanner's cow produces in a day, and let k represent the number of gallons Tony's cow produces in a day. Write an expression to represent how many gallons the family will get from the cows in 30 days. L25, L26

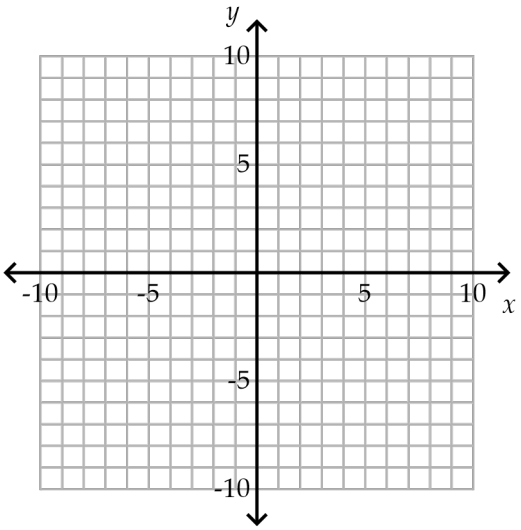
_____

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19. Fill in the table by substituting into the equation, and then use the table to graph the equation. L65, L70

$y = x^2 + 2$

x	y
–2	
–1	
0	
1	
2	



20. Fill in the coordinates of the image if the preimage is reflected over the x-axis. L72

Preimage	$(-4,2)$	$(-9,2)$	$(-4,7)$	$(-9,7)$
Image				

21. The table below represents a function. Identify the rule of the function and fill in the missing value in the table. L69

x	y
0	4
1	5
2	
3	7
4	8

Rule: _____

Missing Value: _____

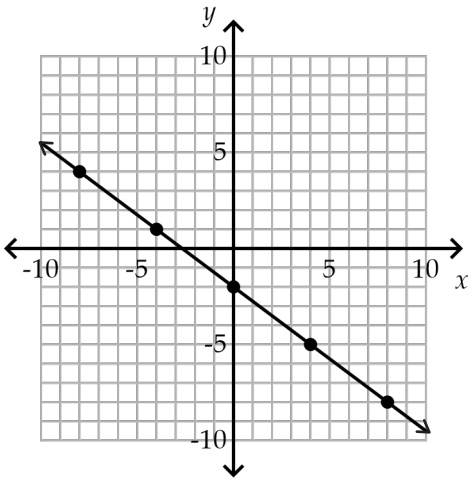
22. Write the letter of the equation that matches the line graphed below. L66–68

a. $y = \frac{1}{3}x + 8$

b. $y = 2x - 3$

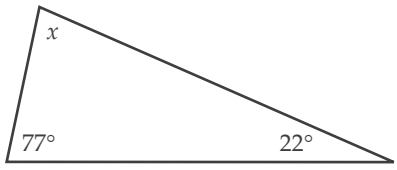
c. $y = -x + 6$

d. $y = -\frac{3}{4}x - 2$



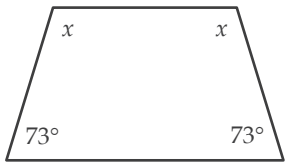
_____

23. Find the measure of the missing angle in the triangle. L71



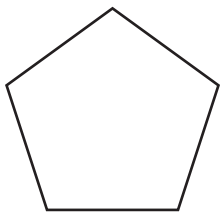
$x =$ _____

24. Find the measure of the missing angles in the isosceles trapezoid. L77



$x =$ _____

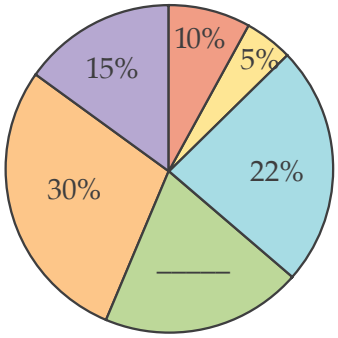
25. Calculate the sum of the interior angles of the pentagon by splitting the shape into the fewest number of triangles possible. L76



_____

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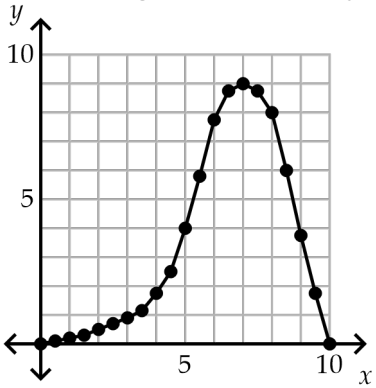
41. Fill in the missing percent in the circle graph below. L104



42. Find the probability (as a fraction) of getting a window seat on an airplane if the seats are randomly assigned and 60 of the 180 seats are window seats. L111

_____

43. Determine whether the graph is left skewed, right skewed, or symmetric. L110



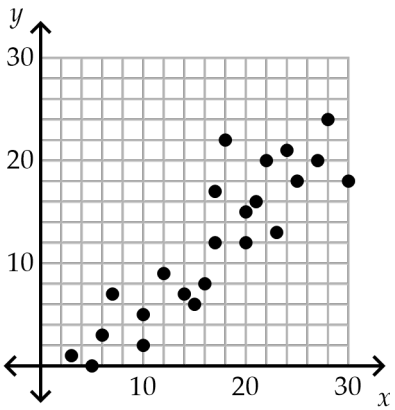
Circle one:

left skewed

right skewed

symmetric

44. Determine the correlation in the scatter plot. Write *positive*, *negative*, or *no correlation* on the line. L109



_____

45. Determine if the following events are mutually exclusive. Write *yes* or *no* on the line. L112

a. Event 1: ordering french fries at a restaurant

Event 2: ordering onion rings at the same restaurant

_____

b. Event 1: winning a race

Event 2: losing the same race

_____

46. Write the data in numerical order. Then find the mean, median, mode, and range of the data set. L106

7, 7, 8, 10, 11, 11, 9, 9, 63, 15, 13, 11, 13, 12, 15, 14

Numerical order: _____

_____

Mean: _____ Median: _____

Mode: _____ Range: _____

47. For the data set in Problem 46, is the mean or median a better measure of center? L107

Circle one: *mean* / *median*

Why? _____

_____

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Enrichment: Patterns with Divisibility

SUPPLIES: 20–30 small objects

Counting manipulatives are needed for this lesson. Any small objects may be used (beans, cereal pieces, buttons, etc.). Do not use more than 30 manipulatives.



This is an enrichment lesson. Mastery is not expected at this level. There is no video, practice, or review. A calculator may be used for this entire lesson.

Divisibility by 3

Avoid using long division to answer questions. Instead, use the manipulatives and rely on patterns to answer the questions about divisibility.

1. How many groups of 3 can be made with 10 items? How many are left over?

   Groups: _____ Remaining: _____

2. How many groups of 3 can be made with 2 tens (two groups of 10)? How many are left over?

   Groups: _____ Remaining: _____

   How does this answer relate to the answer for Question 1?

   _____

3. How many will be left over after making groups of 3 out of 3 groups of 10? Why is this?

   Remaining: _____

   Why?

   _____

   _____

4. How many will be left over after making groups of 3 out of 7 groups of 10? Why is this?

   Note: Do not use manipulatives for problems with more than 30 items.

   Remaining: _____

   Why?

   _____

   _____

5. Are there any remaining when 72 is split into groups of 3? Why is this?

   Remaining: _____

   Why?

   _____

   _____

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UNITS
1-4

Simply Good and Beautiful

MATH 7

ANSWERS & SOLUTIONS

The Good and the Beautiful
CURRICULUM

Note to Parents

Math 7 Answers and Solutions **includes detailed solutions for all problems in the course book. Final answers are boxed for easy reference. Solutions are not included for the Video Notes section of each lesson. Parents do not need to check this section. Students complete notes along with the video instructor and try problems on their own that are then checked in the video. Student answers may not be correct in this section, and that is OK. Mastery is not expected in the Video Notes section.**

Table of Contents

UNIT 1 (Lessons 1–30)..... 2

UNIT 2 (Lessons 31–60).....89

UNIT 3 (Lessons 61–90).....169

UNIT 4 (Lessons 91–120)..... 244

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Writing Decimals, Estimating, and Rounding

WARM-UP

a. $45 \div 15 = 3$

b. $16 \bullet 4 = 64$

c. $56 \div 8 = 7$

PRACTICE

1. a. 19 is close to 20.
Estimate: $20 \div 5 = 4$

b. 47 is close to 45.
Estimate: $45 \div 15 = 3$

2.

Problem	Answer Terminates	Answer Repeats
$8.52 \bullet 4.09$	✓	
$103 \div 3$		✓
$39 \div 3$	✓	
$68.6868 \bullet 4.44$	✓	
$56 \div 3$		✓

3. a. $33 \div 13 = 2.\overline{538461}$
$33 \div 13 \approx 2.5385$

b. $4.56 \bullet 2.6398 = 12.037488$
$4.56 \bullet 2.6398 \approx 12.0375$

c. $8.623 \bullet 5.01 = 43.20123$
$8.623 \bullet 5.01 \approx 43.2012$

4. a. $98 \div 15 = 6.53333\ldots = 6.5\overline{3}$

b. $65 \div 12 = 5.4166\ldots = 5.41\overline{6}$

c. $134 \div 11 = 12.181818\ldots = 12.\overline{18}$

5. a. 34 is close to 35.
Estimate: $35 \div 5 = 7$

b. Since $34 < 35$,
the quotient will be less than the estimate.

6. $\$34 \div 5$

$$\begin{array}{r} 6.8 \\ 5\overline{)34.0} \\ -30 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

$\$6.80$

7. $34 \text{ cm} \div 5$

$$\begin{array}{r} 6.8 \\ 5\overline{)34.0} \\ -30 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

6.8 cm

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8. 34 students ÷ 5

$$
\begin{array}{r}
6.8 \\
5\overline{)34.0} \\
-30 \\
\hline
40 \\
-40 \\
\hline
0
\end{array}
$$

Since there cannot be 0.8 of a car, 7 cars are needed.

7 cars

9.

$65 \div 6$	$11.\overline{5}$	12.6
$63 \div 5$	$115 \div 10$	$38 \div 3$
$10.8\overline{3}$	$1266 \div 100$	$10.\overline{83}$
$11.\overline{56}$	12.66	12.7
$254 \div 20$	11.5	$1145 \div 99$
$104 \div 9$	$12.\overline{6}$	$1073 \div 99$

REVIEW

1. O = 30

5	×	6	=	30
6	×	5	=	30
30	÷	5	=	6
30	÷	6	=	5

2. T = 4, R = 24

4	×	6	=	24
6	×	4	=	24
24	÷	4	=	6
24	÷	6	=	4

3. S = 54, E = 9

6	×	9	=	54
9	×	6	=	54
54	÷	6	=	9
54	÷	9	=	6

4. D = 7, I = 63

7	×	9	=	63
9	×	7	=	63
63	÷	7	=	9
63	÷	9	=	7

5. M = 36

3	×	12	=	36
12	×	3	=	36
36	÷	3	=	12
36	÷	12	=	3

6. H = 22

2	×	11	=	22
11	×	2	=	22
22	÷	2	=	11
22	÷	11	=	2

7. C = 72

8	×	9	=	72
9	×	8	=	72
72	÷	8	=	9
72	÷	9	=	8

8. N = 70

7	×	10	=	70
10	×	7	=	70
70	÷	7	=	10
70	÷	10	=	7

Why didn't the dime roll down the mountain with the nickel?

Because IT HAD MORE CENTS!

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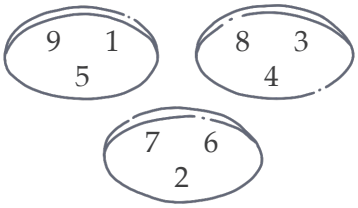
Logic Lesson 1

Logic puzzles can be approached in many different ways. The solutions here may not represent all possible methods or answers.

Hole 1: Answers may vary. An example is given.

$9+8+7+65+4+3+2+1$

Hole 2: Answers may vary. An example is given.



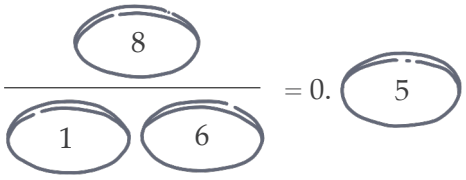
Sum of each hole: 15

Hole 3:

$99 \div 5\frac{1}{2} = 99 \div 5.5 = 18$

18 years old

Hole 4: Answers may vary. An example is given.



8 divided by 16 is 0.5.

Hole 5:

Person A: 8 handshakes

Person B: 7 handshakes (already counted handshake with Person A)

Person C: 6 handshakes (already counted handshakes with Person A and Person B)

Person D: 5 handshakes

Person E: 4 handshakes

Person F: 3 handshakes

Person G: 2 handshakes

Person H: 1 handshake

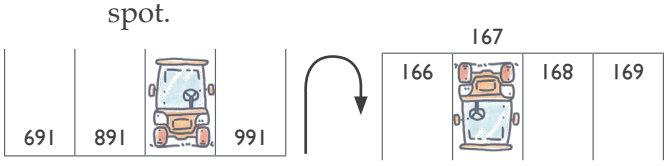
Person I: 0 handshakes (already shook hands with everyone else)

36 handshakes

Hole 6: Answers may vary. An example is given.

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ } \\ 147 \\ 263 \\ +\ 589 \\ \hline 999 \end{array}$$

Hole 7: 167 The numbers are upside down. They are viewed as if pulling into the parking spot.



Hole 8:

1. $9 = 9 \bullet 1$

   $1 __ __ 9$

2. $\frac{2}{\cancel{3}_{1}} \bullet \frac{\overset{3}{\cancel{9}}}{1} = 6$

   $1 __ 6\ 9$

3. $\frac{2}{\cancel{3}_{1}} \bullet \frac{\overset{2}{\cancel{6}}}{1} = 4$

Mystery number: 1469

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Hole 9: Information that can be gathered from each clue is shown below. A check is placed in a box when the answer is known for certain, and an X is placed in a box if it cannot be the answer. When a check is placed in a box, Xs can be placed in the rest of the boxes in the row and column for that section.

		Course				Number of Holes Played			
		Greenfield Golf	Putter's Paradise	Par for the Course	The Cart Club	9	18	27	36
Family	Stewart	X	✓	X	X	✓	X	X	X
	Lin	X	X	X	✓	X	X	X	✓
	Miller	✓	X	X	X	X	✓	X	X
	O'Brien	X	X	✓	X	X	X	✓	X
Number of Holes Played	9	X	✓	X	X				
	18	✓	X	X	X				
	27	X	X	✓	X				
	36	X	X	X	✓				

1. The Millers cannot have played 9 holes because they played more than the Stewarts, and they cannot have played 36 holes because they played less than the O'Briens. The Stewarts cannot have played 36 holes because the Millers played more than them, and the O'Briens did not play 9 holes because the Millers played fewer than them.
2. The family who played Putter's Paradise course cannot have played 36 holes.
3. A check mark can be placed in the box for playing 27 holes at Par for the Course.
4. The O'Briens played 27 or 36 holes.
5. 36 is the only value that is four times another, so the Lins played 36 holes, and the Stewarts played 9 holes. This information, along with Clue 4, means that the Millers played 18 holes and the O'Briens played 27 holes. This means that the O'Briens played Par for the Course (Clue 3).
6. Since the Millers did not play at Putter's Paradise, the Stewarts or Lins must have played this course. However, the family who played Putter's Paradise played 9 or 18 holes. The Lins played 36 holes, so they could not have played Putter's Paradise. The Stewarts played 9 holes, so they played Putter's Paradise.
7. The only options left for Greenfield Golf are 18 or 36 holes. Since the family who played this course played 9 or 18 holes, they must have played 18 holes. Therefore the family who played The Cart Club played 36 holes. The rest of the answers can now be found based on the available options, but two additional clues are given.
8. The Lin family played 36 holes at The Cart Club.
9. The Miller family played at Greenfield Golf.

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Unit 1 Review

1. $200 \bullet 4 = 800$
$250 \bullet 4 = 1000$
Estimate: between 800 and 1000

Rounded:
$241 \bullet 3.7412 = 901.6292 \approx 901.629$

2.
$$\begin{array}{r} 13.16\ldots \\ 48\overline{)632.00\ldots} \\ -48 \\ \hline 152 \\ -144 \\ \hline 80 \\ -48 \\ \hline 320 \\ -288 \\ \hline 32 \end{array}$$

$632 \div 48 = 13.1\overline{6}$

3.

Fraction	Decimal
$\frac{3}{5}$	0.6
$\frac{2}{3}$	$0.\overline{6}$
$4\frac{5}{8}$	4.625

Detailed work is shown below.

$0.6 = \frac{6}{10} = \frac{3}{5}$

$\frac{2}{3} \rightarrow$
$$\begin{array}{r} 0.6\ldots \\ 3\overline{)2.0\ldots} \\ -18 \\ \hline 2 \end{array}$$

$4\frac{5}{8} = \frac{37}{8}$

$$\begin{array}{r} 4.625 \\ 8\overline{)37.000} \\ -32 \\ \hline 50 \\ -48 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

4.
$$\begin{array}{r|l} 2 & 420 \\ 2 & 210 \\ 3 & 105 \\ 5 & 35 \\ & 7 \end{array} \qquad \begin{array}{r|l} 2 & 600 \\ 2 & 300 \\ 2 & 150 \\ 3 & 75 \\ 5 & 25 \\ & 5 \end{array}$$

$\frac{420}{600} = \frac{\cancel{2} \bullet \cancel{2} \bullet \cancel{3} \bullet \cancel{5} \bullet 7}{\cancel{2} \bullet \cancel{2} \bullet 2 \bullet \cancel{3} \bullet \cancel{5} \bullet 5} = \frac{7}{10}$

5.
$$\begin{array}{r|l} 2 & 248 \\ 2 & 124 \\ 2 & 62 \\ & 31 \end{array} \qquad \begin{array}{r|l} 2 & 1240 \\ 2 & 620 \\ 2 & 310 \\ 5 & 155 \\ & 31 \end{array}$$

$\frac{248}{1240} = \frac{\cancel{2} \bullet \cancel{2} \bullet \cancel{2} \bullet \cancel{31}}{\cancel{2} \bullet \cancel{2} \bullet \cancel{2} \bullet 5 \bullet \cancel{31}} = \frac{1}{5}$

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6.

Expression	Value
$4-10+2+7-1$	2
$-3+5-7+4-2$	–3
$-1+4-2-(-1)$	2

Detailed work is shown below.

$4-10+2+7-1$
$=-6+2+7-1$
$=-4+7-1$
$=3-1$
$=2$

$-3+5-7+4-2$
$=2-7+4-2$
$=-5+4-2$
$=-1-2$
$=-3$

$-1+4-2-(-1)$
$=3-2-(-1)$
$=1-(-1)$
$=1+1$
$=2$

Expression	Value
$\frac{3}{4}-\frac{7}{8}$	$-\frac{1}{8}$
$\frac{1}{2}+\left(-\frac{5}{8}\right)$	$-\frac{1}{8}$
$3-6.5$	–3.5

Detailed work is shown below.

$\frac{3}{4}-\frac{7}{8}=\frac{6}{8}-\frac{7}{8}=-\frac{1}{8}$

$\frac{1}{2}+\left(-\frac{5}{8}\right)=\frac{4}{8}+\left(-\frac{5}{8}\right)=-\frac{1}{8}$

$3-6.5=3+(-6.5)$
$6.5-3=3.5$
$|-6.5|>|3|$
$3-6.5=-3.5$

7. a. same sign, positive answer

$$\begin{array}{r} {\scriptstyle 1}\\ 17\\ \times\ 2\\ \hline 34 \end{array}$$

$-17\bullet(-2)=34$

b. different signs, negative answer

$$\begin{array}{r} 3.4\\ \times\ 1.2\\ \hline 6\,8\\ +\ 3\,4\,0\\ \hline 4.08 \end{array}$$

$1.2\bullet(-3.4)=-4.08$

8. a. different signs, negative answer

$$7\overline{)84}\quad 12$$
$$\begin{array}{r} -7\\ \hline 14\\ -14\\ \hline 0 \end{array}$$

$84\div(-7)=-12$

b. different signs, negative answer

$$\frac{-3\frac{4}{5}}{\frac{5}{12}}$$

$=-3\frac{4}{5}\div\frac{5}{12}$
$=-\frac{19}{5}\bullet\frac{12}{5}$
$=-\frac{228}{25}$
$=-9\frac{3}{25}$

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$$\frac{-3\frac{4}{5}}{\frac{5}{12}} = -9\frac{3}{25}$$

9.

Exponent Form	Expansion	Simplified
4^3	$4 \bullet 4 \bullet 4$	64
$(-3)^5$	$-3(-3)(-3)(-3)(-3)$	-243
$\frac{75}{5^4}$	$\frac{75}{5 \bullet 5 \bullet 5 \bullet 5}$	$\frac{3}{25}$
$\left(\frac{4}{5}\right)^4$	$\frac{4}{5} \bullet \frac{4}{5} \bullet \frac{4}{5} \bullet \frac{4}{5}$	$\frac{256}{625}$
3^{-5}	$\frac{1}{3 \bullet 3 \bullet 3 \bullet 3 \bullet 3}$	$\frac{1}{243}$
10^{-6}	$\frac{1}{10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10}$	$\frac{1}{1000000}$

Detailed work is shown below.

$$(-3)^5$$
$$= -3 \bullet (-3) \bullet (-3) \bullet (-3) \bullet (-3)$$
$$= 9 \bullet (-3) \bullet (-3) \bullet (-3)$$
$$= -27 \bullet (-3) \bullet (-3)$$
$$= 81 \bullet (-3)$$
$$= -243$$

$$\frac{75}{5^4} = \frac{75}{5 \bullet 5 \bullet 5 \bullet 5} = \frac{3 \bullet \cancel{5} \bullet \cancel{5}}{\cancel{5} \bullet \cancel{5} \bullet 5 \bullet 5} = \frac{3}{25}$$

$$\left(\frac{4}{5}\right)^4 = \frac{4}{5} \bullet \frac{4}{5} \bullet \frac{4}{5} \bullet \frac{4}{5} = \frac{256}{625}$$

$$10^{-6} = \frac{1}{10^6} = \frac{1}{10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10} = \frac{1}{1000000}$$

10. a. $10^{-2} \rightarrow 1.$
0.01

b. $10^{-5} \rightarrow 1.$
0.00001

11. a.
$$500002014$$
$$= 500000000 + 2000 + 10 + 4$$
$$= (5 \bullet 100000000) + (2 \bullet 1000) + (1 \bullet 10) + (4 \bullet 1)$$
$$= (5 \bullet 10^8) + (2 \bullet 10^3) + (1 \bullet 10^1) + (4 \bullet 10^0)$$

b.
$$6284.203$$
$$= 6000 + 200 + 80 + 4 + 0.2 + 0.003$$
$$= (6 \bullet 1000) + (2 \bullet 100) + (8 \bullet 10) + (4 \bullet 1) + (2 \bullet 0.1) + (3 \bullet 0.001)$$
$$= (6 \bullet 10^3) + (2 \bullet 10^2) + (8 \bullet 10^1) + (4 \bullet 10^0) + (2 \bullet 10^{-1}) + (3 \bullet 10^{-3})$$

12. a. $54{,}382{,}300{,}000 = 5.43823 \times 10^{10}$

b. $0.0000000423 = 4.23 \times 10^{-8}$

13. a. $8.15 \times 10^{-3} \rightarrow 8.15$
0.00815

b. $9.01 \times 10^6 \rightarrow 9.01$
9010000

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Unit 1 Assessment

Note to parent/teacher: This assessment covers concepts taught in Unit 1. Problems are designed to assess multiple skills. If a problem is missed, show the student the answer and allow him or her time to find the error. Often, students can correct mistakes when checking their work. If the student still has difficulty, have him or her revisit the corresponding lesson for review. Corresponding lesson numbers are listed in the course book at the end of each problem.

1. a. $7 \cdot 3 = 21$

$8 \cdot 3 = 24$

Estimate: between 21 and 24

Rounded: $7.412 \cdot 3 = 22.236 \approx 22.24$

b. $6 \cdot 4 = 24$

Estimate: 24

Rounded: $5.963 \cdot 4.2 = 25.0446 \approx 25.04$

2. a. $2 \overline{)342}$

$3 \overline{)171}$

$3 \overline{)57}$

19

$342 = 2 \cdot 3^2 \cdot 19$

b. $2 \overline{)450}$

$3 \overline{)225}$

$3 \overline{)75}$

$5 \overline{)25}$

5

$450 = 2 \cdot 3^2 \cdot 5^2$

3. $\frac{342}{450} = \frac{\cancel{2} \cdot \cancel{3} \cdot \cancel{3} \cdot 19}{\cancel{2} \cdot \cancel{3} \cdot \cancel{3} \cdot 5 \cdot 5} = \frac{19}{25}$

4. $420 \overline{)70} \longrightarrow 42 \overline{)7.00...}$

$-4\,2$

$2\,80$

$-2\,52$

28

$70 \div 420 = 0.\overline{16}$

5. a.
$$\begin{array}{r} 0.8 \\ 5\overline{)4.0} \\ -4\,0 \\ \hline 0 \end{array}$$

$\frac{4}{5} = 0.8$

b.
$$\begin{array}{r} 0.83... \\ 6\overline{)5.00...} \\ -4\,8 \\ \hline 20 \\ -18 \\ \hline 2 \end{array}$$

$\frac{5}{6} = 0.8\overline{3}$

6. a. $0.125 = \frac{125}{1000} = \frac{1}{8}$

b. $5.43 = 5\frac{43}{100}$

7. a. $25 - 38 + 15 = -13 + 15 = 2$

b. $-7 - 33 - 32 = -40 - 32 = -72$

c. $67 - 35 + 12 = 32 + 12 = 44$

8. a. same signs, positive $-14 \cdot (-3) = 42$

b. different signs, negative $-90 \div 10 = -9$

c. different signs, negative $21 \cdot (-5) = -105$

d. same signs, positive $-55 \div (-11) = 5$

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9. a. $-\frac{\cancel{7}^{1}}{8} \bullet \left(-\frac{3}{\cancel{14}_{2}}\right) = \frac{3}{16}$

b. $-4\frac{5}{6} \div \frac{2}{9}$

$= -\frac{29}{6} \div \frac{2}{9}$

$= -\frac{29}{\cancel{6}_{2}} \bullet \frac{\cancel{9}^{3}}{2}$

$= -\frac{87}{4}$

$= -21\frac{3}{4}$

10. $\frac{\frac{4}{15}}{\frac{8}{45}}$

$= \frac{4}{15} \div \frac{8}{45}$

$= \frac{\cancel{4}^{1}}{\cancel{15}_{1}} \bullet \frac{\cancel{45}^{3}}{\cancel{8}_{2}}$

$= \frac{3}{2}$

$= 1\frac{1}{2}$

11. a. $\frac{5}{7} + 3\frac{11}{14} - \frac{25}{28}$

$= \frac{5}{7} + \frac{53}{14} - \frac{25}{28}$

$= \frac{20}{28} + \frac{106}{28} - \frac{25}{28}$

$= \frac{126}{28} - \frac{25}{28}$

$= \frac{101}{28}$

$= 3\frac{17}{28}$

b. $-\frac{3}{2} - \frac{8}{11} + \frac{1}{11}$

$= -\frac{33}{22} - \frac{16}{22} + \frac{2}{22}$

$= -\frac{49}{22} + \frac{2}{22}$

$= -\frac{47}{22}$

$= -2\frac{3}{22}$

12. a.
$$\begin{array}{r} 4.3570 \\ 10.3210 \\ +\ 5.0101 \\ \hline 19.6881 \end{array}$$

b.
$$\begin{array}{r} 12.987 \\ -\ 4.012 \\ \hline 8.975 \end{array} \longrightarrow 8.975 - 9.754$$

Subtract and use the sign of the greater absolute value.

$$\begin{array}{r} \cancel{9}^{8}\cancel{.}^{16}\cancel{7}^{14}\cancel{5}^{1}4 \\ -8.9\,7\,5 \\ \hline 0.7\,7\,9 \end{array}$$

The answer is negative.

$12.987 - 4.012 - 9.754 = -0.779$

13. a.
$$\begin{array}{r} \cancel{1}^{1} \\ 2.3 \\ \times\ 4.5 \\ \hline 115 \\ +\ 920 \\ \hline 10.35 \end{array}$$

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Enrichment: Sequences and Series

This is an enrichment lesson. Students are not expected to master content in the enrichment lessons at this level.

1.

	Rule	Next Two Terms	A, G, or N	Common Difference or Ratio
a.	+2	14, 16	A	2
b.	perfect squares	49, 64	N	none
c.	–6	70, 64	A	–6
d.	÷2	$\frac{1}{4}, \frac{1}{8}$	G	$\frac{1}{2}$
e.	+2	15, 17	A	2
f.	•1.5	60.75, 91.125	G	1.5
g.	prime numbers	19, 23	N	none
h.	•2	192, 384	G	2

2. a. The sequence is arithmetic with a common difference of 6.

$$\begin{aligned} a_{17} &= 5+(17-1)(6) \\ &= 5+(16)(6) \\ &= 5+(96) \\ &= 101 \end{aligned}$$

b. The sequence is arithmetic with a common difference of –3.

$$\begin{aligned} a_{42} &= 7+(42-1)(-3) \\ &= 7+(41)(-3) \\ &= 7+(-123) \\ &= -116 \end{aligned}$$

c. The sequence is arithmetic with a common difference of 3.5.

$$\begin{aligned} a_{101} &= -10+(101-1)(3.5) \\ &= -10+(100)(3.5) \\ &= -10+(350) \\ &= 340 \end{aligned}$$

3. a. The sequence is geometric with a common ratio of 2.

$$\begin{aligned} b_{11} &= 1 \bullet 2^{11-1} \\ &= 1 \bullet 2^{10} \\ &= 1 \bullet 1024 \\ &= 1024 \end{aligned}$$

b. The sequence is geometric with a common ratio of $\frac{1}{3}$.

$$\begin{aligned} b_9 &= 729 \bullet \left(\frac{1}{3}\right)^{9-1} \\ &= 729 \bullet \left(\frac{1}{3}\right)^{8} \\ &= 729 \bullet \frac{1}{6561} \\ &= \frac{1}{9} \end{aligned}$$

c. This sequence is geometric with a common ratio of $\frac{3}{2}$.

$$\begin{aligned} b_6 &= 18 \bullet \left(\frac{3}{2}\right)^{6-1} \\ &= 18 \bullet \left(\frac{3}{2}\right)^{5} \\ &= 18 \bullet \frac{243}{32} \\ &= 136.6875 \end{aligned}$$

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Multi-Step Equations with Negative Coefficients

WARM-UP

1. a. $8^2 = 8 \bullet 8 = 64$
   b. $9^2 = 9 \bullet 9 = 81$
   c. $\sqrt{16} = 4$
   d. $\sqrt{25} = 5$

2. a. $120 \bullet 4 = 480$
   b. $800 \bullet 12 = 9600$

PRACTICE

1. **Blue**

$$-3x + 4 = 20 - x$$
$$-3x + x + 4 = 20 - x + x$$
$$-2x + 4 = 20$$
$$-2x + 4 - 4 = 20 - 4$$
$$-2x = 16$$
$$\frac{-2x}{-2} = \frac{16}{-2}$$
$$x = -8$$

2. **Green**

$$3 - 2a + 5 = -12 + 2a$$
$$3 - 2a + 5 + 2a = -12 + 2a + 2a$$
$$8 = -12 + 4a$$
$$8 + 12 = -12 + 4a + 12$$
$$20 = 4a$$
$$\frac{20}{4} = \frac{4a}{4}$$
$$5 = a$$
$$a = 5$$

3. **Yellow**

$$7 - 2b = 1 + b + 5 - 2b$$
$$7 - 2b = 1 + 5 - b$$
$$7 - 2b = 6 - b$$
$$7 - 2b + b = 6 - b + b$$
$$7 - b = 6$$
$$7 - b - 7 = 6 - 7$$
$$-b = -1$$
$$\frac{-b}{-1} = \frac{-1}{-1}$$
$$b = 1$$

4. **Orange**

$$12 + 14s = 72 - 6s$$
$$12 + 14s + 6s = 72 - 6s + 6s$$
$$12 + 20s = 72$$
$$12 + 20s - 12 = 72 - 12$$
$$20s = 60$$
$$\frac{20s}{20} = \frac{60}{20}$$
$$s = 3$$

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5. **Red**

$$3r-1=r+3$$
$$3r-1-r=r+3-r$$
$$2r-1=3$$
$$2r-1+1=3+1$$
$$2r=4$$
$$\frac{2r}{2}=\frac{4}{2}$$
$$r=2$$

6. **Blue**

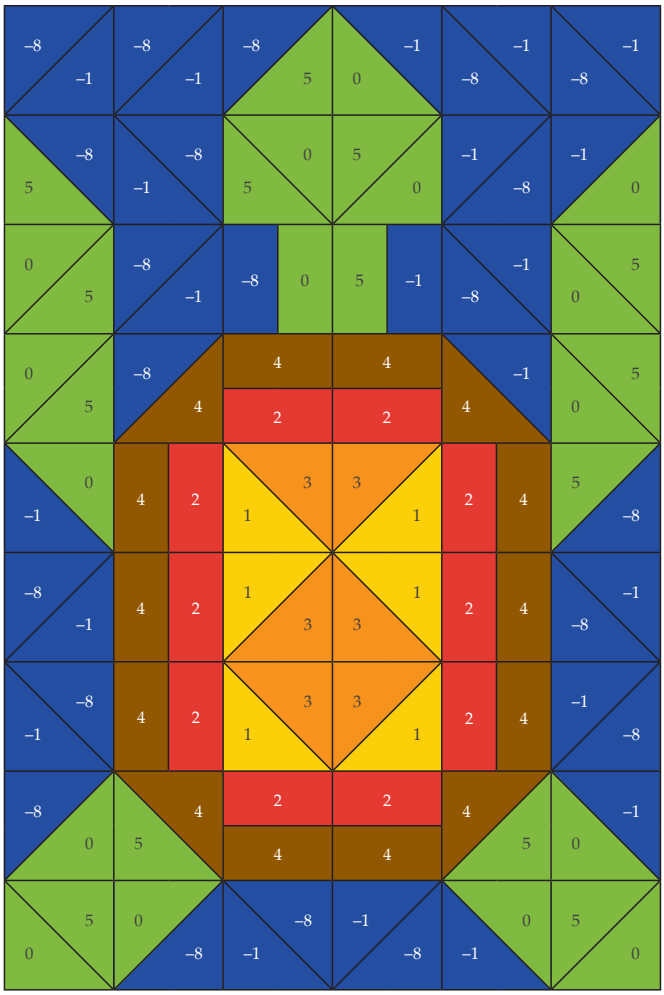
$$4y+5=7+y-5$$
$$4y+5=y+2$$
$$4y+5-y=y+2-y$$
$$3y+5=2$$
$$3y+5-5=2-5$$
$$3y=-3$$
$$\frac{3y}{3}=\frac{-3}{3}$$
$$y=-1$$

7. **Green**

$$2-3z=z+2$$
$$2-3z+3z=z+2+3z$$
$$2=4z+2$$
$$2-2=4z+2-2$$
$$0=4z$$
$$\frac{0}{4}=\frac{4z}{4}$$
$$0=z$$
$$z=0$$

8. **Brown**

$$p-3=5-p$$
$$p-3+p=5-p+p$$
$$2p-3=5$$
$$2p-3+3=5+3$$
$$2p=8$$
$$\frac{2p}{2}=\frac{8}{2}$$
$$p=4$$



Mixed Review

Tables may vary. Students must have the five required ingredients in their tables (tortillas, meat, cheese, beans, sauce). The total cost must be $35.00 or less. A sample table with three extra ingredients is shown below. Check the student's work using a calculator.

Ingredient	Variety	Unit Cost	Quantity	Ingredient Cost
Tortillas	Precooked flour	$2.75	1	$2.75 • 1 = $2.75
Meat	Pork	$4.25	2	$4.25 • 2 = $8.50
Cheese	3-Cheese blend	$5.78	1	$5.78 • 1 = $5.78
Beans	Pinto beans	$0.78	3	$0.78 • 3 = $2.34
Sauce	Green sauce	$2.19	3	$2.19 • 3 = $6.57
Extra Ingredient 1	Onion	$0.65	1	$0.65 • 1 = $0.65
Extra Ingredient 2	Sour cream	$2.48	1	$2.48 • 1 = $2.48
Extra Ingredient 3	Red sauce	$1.78	1	$1.78 • 1 = $1.78
Extra Ingredient 4				
Extra Ingredient 5				
			Subtotal	$30.85
			Tax Amount (3%)	$0.93
			TOTAL	**$31.78**

Subtotal: $2.75 + $8.50 + $5.78 + $2.34 + $6.57 + $0.65 + $2.48 + $1.78 = $30.85

Tax Amount: 0.03 • $30.85 ≈ $0.93

TOTAL: $30.85 + $0.93 = $31.78

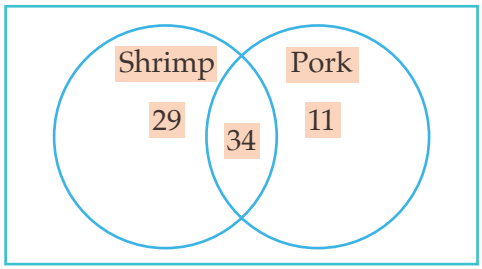
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Unit 2 Review

1. a. The number of people who want both shrimp and pork goes in the overlapping region.

Subtract the number of people who want both from the number of people who want shrimp.
$63 - 34 = 29$

Subtract the number of people who want both from the number of people who want pork.
$45 - 34 = 11$

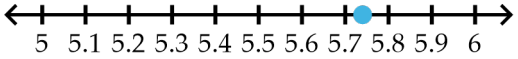


b. $29 + 34 + 11 = 74$ villagers

c. $A \cap B$

2. a. 33 is between the perfect squares 25 and 36.

b. $\sqrt{33} \approx 5.74$

c. 

3. a. $\sqrt{81} = 9$

b. $\sqrt[3]{64} = 4$

c. $\sqrt[3]{-27} = -3$

4. a.
$$\frac{2}{3}a - 1 = 1\frac{1}{3}$$
$$\frac{2}{3}a - 1 + 1 = 1\frac{1}{3} + 1$$
$$\frac{2}{3}a = \frac{4}{3} + \frac{3}{3}$$
$$\frac{2}{3}a = \frac{7}{3}$$
$$\frac{\cancel{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{\cancel{3}}a = \frac{7}{\cancel{3}} \cdot \frac{\cancel{3}}{2}$$
$$a = \frac{7}{2}$$
$$a = 3\frac{1}{2}$$

b.
$$1 - 0.7b = -0.75$$
$$1 - 0.7b - 1 = -0.75 - 1$$
$$-0.7b = -1.75$$
$$\frac{-0.7b}{-0.7} = \frac{-1.75}{-0.7}$$
$$b = 2.5$$

5. a. $37 = 5m + 2$

b.
$$37 = 5m + 2$$
$$37 - 2 = 5m + 2 - 2$$
$$35 = 5m$$
$$\frac{35}{5} = \frac{5m}{5}$$
$$7 = m$$
$$m = 7$$

7 mud carp

6.
$$C = \pi d$$
$$\frac{C}{\pi} = \frac{\pi d}{\pi}$$
$$\frac{C}{\pi} = d$$
$$d = \frac{C}{\pi}$$

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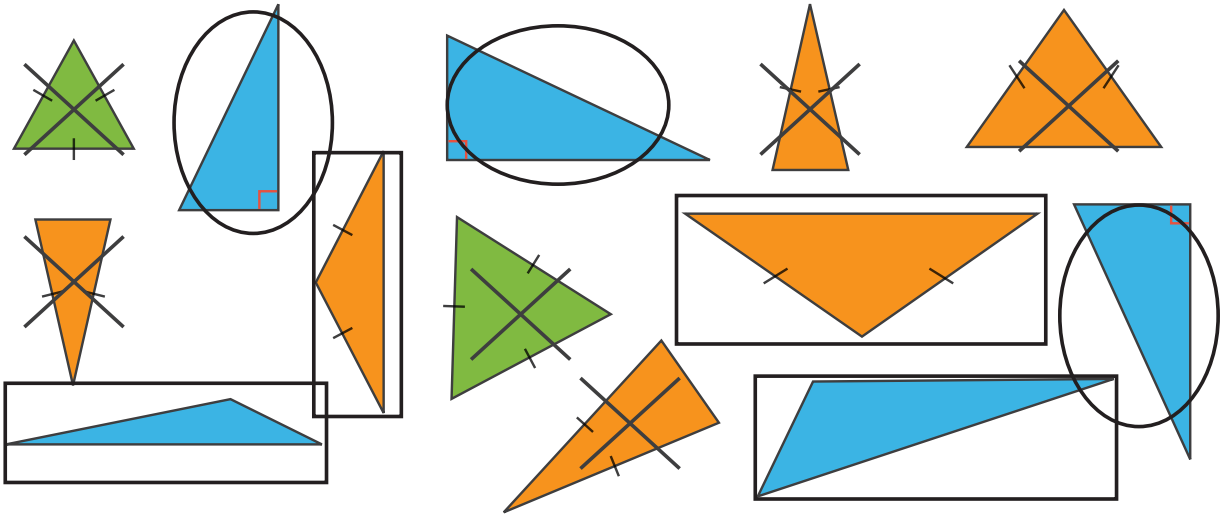
Triangles

WARM-UP

$$
\begin{aligned}
2p + 4p - 3p &= 21 \\
6p - 3p &= 21 \\
3p &= 21 \\
\frac{3p}{3} &= \frac{21}{3} \\
p &= 7
\end{aligned}
$$

PRACTICE

1.



2.

A triangle has two angles measuring 40°. What is the measure of the third angle?	74°	**A** A right triangle has an angle measuring 36°. What is the measure of the third angle?	80°
60°	**B** What are each of the angle measures in an equilateral triangle?	**C** An isosceles triangle has one angle that measures 20°. What is the measure of each of the two congruent angles?	**D** What is the measure of each of the non-right angles of an isosceles right triangle?
45°	**E** A triangle has two angles measuring 35° and 71°. What is the third angle measure?	100°	54°

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Detailed work for the problems is shown below.

A
$$\begin{aligned} 90° + 36° + x &= 180° \\ 126° + x &= 180° \\ 126° + x - 126° &= 180° - 126° \\ x &= 54° \end{aligned}$$

B
$$\begin{aligned} x + x + x &= 180° \\ 3x &= 180° \\ \frac{3x}{3} &= \frac{180°}{3} \\ x &= 60° \end{aligned}$$

C
$$\begin{aligned} x + x + 20° &= 180° \\ 2x + 20° &= 180° \\ 2x + 20° - 20° &= 180° - 20° \\ 2x &= 160° \\ \frac{2x}{2} &= \frac{160°}{2} \\ x &= 80° \end{aligned}$$

D
$$\begin{aligned} x + x + 90° &= 180° \\ 2x + 90° &= 180° \\ 2x + 90° - 90° &= 180° - 90° \\ 2x &= 90° \\ \frac{2x}{2} &= \frac{90°}{2} \\ x &= 45° \end{aligned}$$

E
$$\begin{aligned} 35° + 71° + x &= 180° \\ 106° + x &= 180° \\ x + 106° - 106° &= 180° - 106° \\ x &= 74° \end{aligned}$$

3. a. $3 + 4 = 7$ 7 is greater than 5 (the third side).
$4 + 5 = 9$ 9 is greater than 3.
$3 + 5 = 8$ 8 is greater than 4.
yes

b. $3 + 3 = 6$ 6 is greater than 5.
$3 + 5 = 8$ 8 is greater than 3.
yes

c. $4 + 6 = 10$ 10 is not greater than the third side of 11.
no

REVIEW

1. a. $4 \div 2 = 2$
$6 \div 4 = 1.5$
$8 \div 6 = 1.\overline{3}$
no

b. $10 \div 2 = 5$
$35 \div 7 = 5$
$55 \div 11 = 5$
yes

2.
$$\begin{array}{r} \overset{10}{\cancel{11}}\ \text{min}\ \overset{73}{\cancel{13}}\ \text{sec} \\ -\ 9\ \text{min}\ 49\ \text{sec} \\ \hline 1\ \text{min}\ 24\ \text{sec} \end{array}$$

3. 4 child passes + 9 adult passes = 13 total passes

$$\begin{aligned} \frac{4 \text{ child passes}}{13 \text{ total passes}} &= \frac{144 \text{ child passes}}{x \text{ total passes}} \\ \frac{4}{13} &= \frac{144}{x} \\ 4x &= 13 \bullet 144 \\ 4x &= 1872 \\ \frac{4x}{4} &= \frac{1872}{4} \\ x &= 468 \end{aligned}$$

468 total passes

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Enrichment: Circumference and Diameter

This is an enrichment lesson. Students are not expected to master content in the enrichment lessons at this level.

Part 1

Items 1, 2, and 3: Answers may vary for students' measured circles. An example is given below.

Item 1: top of a mug

Circumference: 11 in

Diameter: 3.5 in

Ratio: $\frac{C}{d} = \frac{11}{3.5} \approx 3.1429$

1. Students may notice that the ratio quotients are very close to each other. Students may also notice that the quotient is close to the value of pi.

2. $C = \pi d$

$\frac{C}{d} = \frac{\pi d}{d}$

$\frac{C}{d} = \pi$

$\pi = \frac{C}{d}$

Part 2

First Circle:

2. a. Radius: 3 in
   b. Diameter: 6 in
   c. Circumference: 6π in

12. a. The radius is approximately the same length as BC.
   b. Half of the circumference is approximately the same length as AB.

Second Circle:

8. Students may now notice that AB is approximately half of the circumference, and BC is approximately the length of the radius.

Third Circle:

8. Half of the circumference is colored in each color.

9. Side AB is approximately half of the circumference of the circle.

10. 3 inches (It is the radius of the circle.)

11. Side BC is approximately the length of the radius of the circle.

12. a. Length: $AB \approx \frac{1}{2}C$
   b. Width: $BC \approx r$
   c. $A = lw = AB \bullet BC \approx \frac{1}{2}C \bullet r$

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Simplifying Rational Expressions

WARM-UP

a. $\frac{64}{16}$

$= \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2}}$

$= \frac{4}{1} = 4$

b. $\frac{24}{30}$

$= \frac{\cancel{2} \cdot 2 \cdot 2 \cdot \cancel{3}}{\cancel{2} \cdot \cancel{3} \cdot 5}$

$= \frac{4}{5}$

c. $\frac{9}{54}$

$= \frac{\cancel{3} \cdot \cancel{3}}{\cancel{3} \cdot \cancel{3} \cdot 3 \cdot 2}$

$= \frac{1}{6}$

PRACTICE

9. $\frac{2c}{3a}$

$= \frac{2 \cdot \cancel{b} \cdot \cancel{b} \cdot c \cdot \cancel{a}}{3 \cdot \cancel{a} \cdot a \cdot \cancel{b} \cdot \cancel{b}}$

$\frac{2b^2ca}{3a^2b^2}$

10. $2a$

$= \frac{\cancel{b} \cdot \cancel{a} \cdot a \cdot \cancel{b}^2}{\cancel{b} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{b}}$

$\frac{6a^2b}{3ab}$

11. $3c$

$= \frac{\cancel{a} \cdot \cancel{b} \cdot c}{\cancel{a} \cdot \cancel{b} \cdot \cancel{b}}$

$\frac{9abc}{3ab}$

12. $\frac{a}{bc}$

$= \frac{\cancel{a} \cdot \cancel{b} \cdot \cancel{c} \cdot \cancel{c} \cdot \cancel{c} \cdot \cancel{c} \cdot \cancel{c} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{b} \cdot \cancel{b} \cdot \cancel{b} \cdot \cancel{b}}{\cancel{c} \cdot \cancel{c} \cdot \cancel{c} \cdot \cancel{c} \cdot \cancel{c} \cdot \cancel{c} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{b} \cdot \cancel{b} \cdot \cancel{b} \cdot \cancel{b} \cdot \cancel{b} \cdot \cancel{b}}$

$\frac{a^2b^5}{a^6c^6b^6}$

13. $\frac{b}{3c^2}$

$= \frac{\cancel{2} \cdot \cancel{b} \cdot b \cdot \cancel{a}}{\cancel{2} \cdot \cancel{b} \cdot c \cdot \cancel{a} \cdot \cancel{b}}$

$\frac{b \cdot a}{c \cdot ab}$

14. $\frac{3}{b}$

$= \frac{\cancel{y} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{y} \cdot \cancel{y} \cdot b \cdot \cancel{y} \cdot \cancel{y}}{\cancel{x} \cdot \cancel{x} \cdot \cancel{y} \cdot \cancel{y} \cdot \cancel{y} \cdot \cancel{y} \cdot \cancel{y}}$

$\frac{3b^2ac^2}{b^3c^2a}$

15. $6a$

$= \frac{6 \cdot \cancel{b} \cdot \cancel{b} \cdot a}{\cancel{b} \cdot \cancel{b}}$

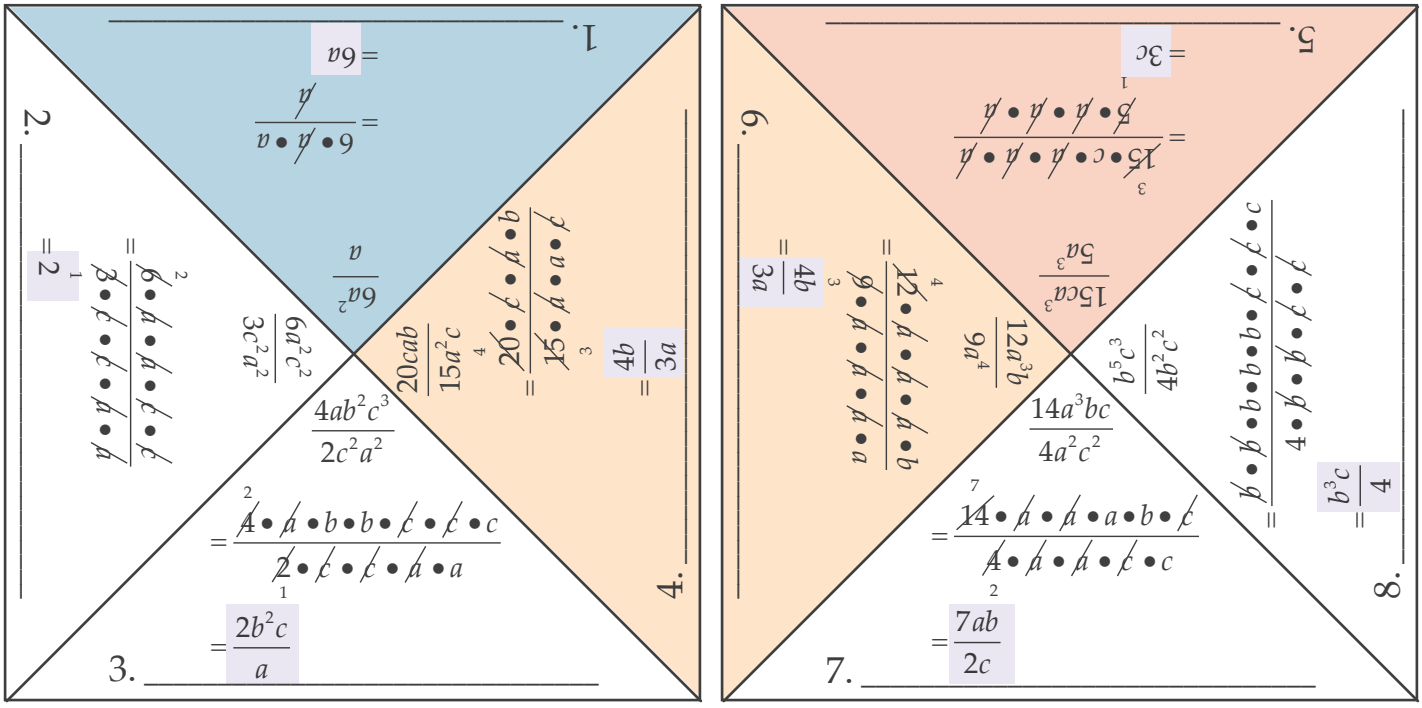
$\frac{6ba^2}{ab}$

16. $2a$

$= \frac{\cancel{2} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{2}}$

$\frac{4ac}{2c}$

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REVIEW

1. a. $6b + 8m + 10c$

   b. $b = 4$, $m = 3$, $c = 10$

   Amount of money made:

   $6(4) + 8(3) + 10(10)$

   $= 24 + 24 + 100$

   $= 148$

   $148

2. Proportion: $\frac{2}{1} = \frac{x}{4}$

   $1 \bullet x = 2 \bullet 4$

   $x = 8$

   Solution: 8 national parks

3. $\left(\frac{1}{50}\right)^2 = \frac{1}{2500}$

4. $V = \pi r^2 h$

   $V = \pi(50)^2(40)$

   $V = \pi \bullet 2500 \bullet 40$

   $V \approx 314159.27$

   $314,159.27 \text{ ft}^3$

5. a. $33°$

   b. $180° - 33° = 147°$

   c. $180° - 33° - 104° = 43°$

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Course Assessment

Note to parent/teacher: This assessment covers concepts taught throughout the course. Corresponding lesson numbers are listed in the course book at the end of each problem.

1. $\frac{224}{672} = \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{7}}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 3 \cdot \cancel{7}} = \frac{1}{3}$

2. a.
$$\begin{array}{r} 0.125 \\ 8\overline{)1.000} \\ -8 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

b.
$$\begin{array}{r} 0.015 \\ 200\overline{)3.000} \\ -200 \\ \hline 1000 \\ -1000 \\ \hline 0 \end{array}$$

3. a. $0.145 = \frac{145}{1000} = \frac{29}{200}$

b. $0.08 = \frac{8}{100} = \frac{2}{25}$

4. a. $5^3 = 5 \cdot 5 \cdot 5 = 125$

b. $\left(\frac{1}{2}\right)^3 = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

c. $3^{-4} = \frac{1}{3^4} = \frac{1}{3 \cdot 3 \cdot 3 \cdot 3} = \frac{1}{81}$

5. a. $\left(5 + 11^2\right) \div 3$
$= \left(5 + 121\right) \div 3$
$= 126 \div 3$
$= 42$

b. $6 \cdot 4 - 19 + 2^3$
$= 6 \cdot 4 - 19 + 8$
$= 24 - 19 + 8$
$= 5 + 8$
$= 13$

6. a. $17z - 4z + 16 + 13$
$= 13z + 29$

b. $4t^2 + 3s - 7t^2 + 5s$
$= 4t^2 - 7t^2 + 3s + 5s$
$= -3t^2 + 8s$

7. a. $\frac{10(3)}{3(-5)} = \frac{30}{-15} = -2$

b. $3(3) - 4(-5) = 9 + 20 = 29$

8. $30m + 30k$

9. a. $12x - 5 = 43$
$12x - 5 + 5 = 43 + 5$
$12x = 48$
$\frac{12x}{12} = \frac{48}{12}$
$x = 4$

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b. $\frac{3}{5}v+4=13-3v$

$\frac{3}{5}v+4-4=13-3v-4$

$\frac{3}{5}v=9-3v$

$\frac{3}{5}v+3v=9-3v+3v$

$\frac{3}{5}v+\frac{15}{5}v=9$

$\frac{18}{5}v=9$

$\frac{5}{18}\bullet\frac{18}{5}v=9\bullet\frac{5}{18}$

$v=\frac{5}{2}=2\frac{1}{2}$

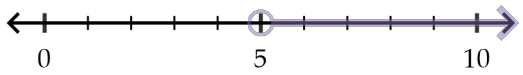
10. a. $11r-3>52$

$11r-3+3>52+3$

$11r>55$

$\frac{11r}{11}>\frac{55}{11}$

$r>5$



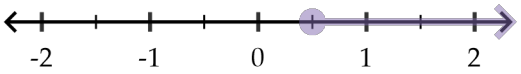
b. $18-4p\le 16$

$18-4p-18\le 16-18$

$-4p\le -2$

$\frac{-4p}{-4}\le\frac{-2}{-4}$

$p\ge\frac{1}{2}$



11. 3 chickens + 2 goats = 5 total

$\frac{5\text{ total}}{2\text{ goats}}=\frac{15\text{ total}}{x\text{ goats}}$

$\frac{5}{2}=\frac{15}{x}$

$5x=30$

$\frac{5x}{5}=\frac{30}{5}$

$x=6$

6 goats

12. $0.4\bullet 320=128$

13. $x\bullet 144=18$

$\frac{x\bullet 144}{144}=\frac{18}{144}$

$x=0.125$

12.5%

14. Amount of increase:

$1350-1000=350$

$x\bullet 1000=350$

$\frac{x\bullet 1000}{1000}=\frac{350}{1000}$

$x=0.35$

35%

15. 1 L = 1,000 mL

$18000\div 1000=18$

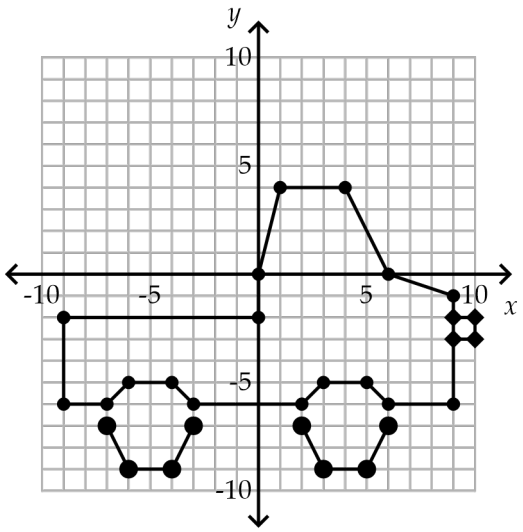
18 liters

16. $8640\text{ in}\bullet\text{in}\bullet\frac{1\text{ft}}{12\text{ in}}\bullet\frac{1\text{ ft}}{12\text{ in}}=\frac{8460\text{ ft}\bullet\text{ft}}{144}=60\text{ ft}^2$

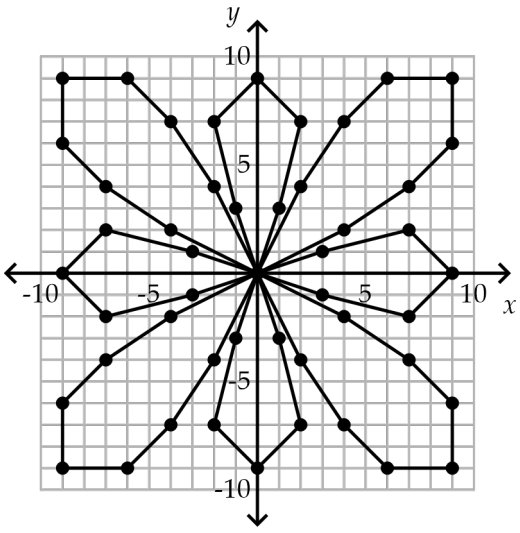
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Fun with Graphing

1.



2.



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