LESSONS
61–90

UNIT
3

Simply Good and Beautiful

PRE-ALGEBRA

COURSE BOOK 3

MATH
8



The Good and the Beautiful
CURRICULUM

COURSE BOOK 3

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Unit 3 Overview

LESSONS 61–90

CONCEPTS COVERED

- Angle relationships
- Arcs and sectors
- Area of circles
- Area of composite figures
- Area of polygons
- Classifying quadrilaterals
- Classifying triangles by angles and sides
- Complementary events
- Compound interest
- Compound probability
- Congruent geometric figures
- Constructing angle bisectors
- Constructing perpendicular bisectors
- Converting between systems of measurement
- Converting units of area
- Converting within systems of measurement
- Convex and concave polygons
- Cross sections
- Drawing triangles given three angle measures
- Drawing triangles given three side lengths
- Experimental probability
- Finding a dimension given volume
- Finding arc length
- Finding missing angle measures
- Finding missing angle measures in similar figures
- Finding missing angles in a circle
- Finding missing interior angles in polygons
- Finding missing side lengths in similar figures
- Finding missing side lengths in similar figures given perimeters and areas
- Finding the area of a sector
- Identifying solutions to inequalities
- Independent and dependent events
- Measuring and drawing angles
- Metric system
- Mutually exclusive events
- Naming angles
- Nets of cones and cylinders
- Nets of prisms and pyramids
- Operations with mixed measures
- Outcomes and sample space
- Parallel lines cut by a transversal
- Percent decrease
- Percent increase
- Points, lines, planes, line segments, rays
- Polygon angles and diagonals
- Properties of polyhedra
- Proportions
- Rates with proportions
- Ratios
- Regular and irregular polygons
- Scale factors
- Scale factors with area
- Scales and scale drawings
- Similar figures
- Simple interest
- Simple probability
- Solving one-step inequalities
- Solving proportions using cross products
- Solving two-step inequalities
- Supplementary and complementary angles
- Surface area of cones, cylinders, and spheres
- Surface area of polyhedra
- Theoretical probability
- Triangle angle sum theorem
- Triangle congruence tests
- Triangle inequality theorem
- Triangle similarity tests
- Types of angles
- Unit multipliers
- Unit rates
- US customary system
- Using area to find missing values
- Volume of composite solids
- Volume of incomplete solids
- Volume of prisms and cylinders
- Volume of pyramids, cones, and spheres

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Unit Conversions and Unit Multipliers

WARM-UP

Convert the measurements in the metric system.

a. 140 m = _____ km b. 5000 daL = _____dL c. 0.008 cg = _____ hg

LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Practice section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

App Video

VIDEO PRACTICE

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LESSON OVERVIEW

Unit multipliers are used to convert between different units of measure while maintaining equality. Converting within a system of measurement (example: feet to yards) can be done quickly by multiplying or dividing by the conversion factor, as shown in the previous lesson. However, unit multipliers can also be used. Unit multipliers are particularly helpful when converting *between* systems of measurement, such as converting from miles to kilometers.

UNIT MULTIPLIERS

A ***unit multiplier*** is a fraction that has units and is equal to 1. Some examples are below.

$$\frac{12 \text{ in}}{1 \text{ ft}} \qquad \frac{4 \text{ qt}}{1 \text{ gal}} \qquad \frac{5280 \text{ ft}}{1 \text{ mi}} \qquad \frac{16 \text{ oz}}{1 \text{ lb}} \qquad \frac{2000 \text{ lb}}{1 \text{ t}}$$

Each fraction above is equal to 1 because the quantity in the numerator and the quantity in the denominator represent the same amount. The reciprocal of a unit multiplier is also a unit multiplier.

To convert units using unit multipliers, multiply the original measurement by a unit multiplier that has the current unit in the denominator. Remember, when a number or value is in the numerator and denominator of a fraction, it will cancel. For example, 345 feet can be converted to yards using a conversion factor that has both feet and yards in it: 3 feet = 1 yard. Writing the unit multiplier with 3 feet in the denominator will allow the unit of feet to cancel. The resulting unit is yards.

$$345 \text{ ft} \cdot \frac{1 \text{ yd}}{3 \text{ ft}} = \frac{345 \text{ yd}}{3} = 115 \text{ yd}$$

REMEMBER: Any whole number can be considered to be in the numerator of a fraction. Multiply fractions straight across. Multiplying by 1 does not have to be shown.

This process works because multiplying by a unit multiplier does not change the value of the original measurement. A unit multiplier is equal to 1, and multiplying by 1 does not change the value of something. Note: The above conversion could have been done by dividing 345 by 3 because there are 3 feet in 1 yard, but it is helpful to learn how unit multipliers work through simple examples.

Often, more than one unit multiplier is needed for a conversion. If conversion factors between the original unit and ending unit are not known, use known conversion factors as stepping stones. Write the unit multiplier so the original unit is in the denominator. Then write the next unit multiplier with the new unit in the denominator. In this way, units will continue to cancel until the desired unit is left.

Example 1: Convert 3.5 miles to inches.

Note: The conversion factor from miles to inches is not commonly known, but the conversion factor from miles to feet is. A unit multiplier can be used to convert from miles to feet and then from feet to inches.

$$3.5 \text{ mi} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} = \frac{3.5 \cdot 5280 \cdot 12 \text{ in}}{1} = 221{,}760 \text{ in}$$

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Example 2: Convert 2 tons to ounces.

$$2\ \cancel{t} \bullet \frac{2000\ \cancel{lb}}{1\ \cancel{t}} \bullet \frac{16\ oz}{1\ \cancel{lb}} = \frac{2 \bullet 2000 \bullet 16\ oz}{1} = 64{,}000\ oz$$

If a conversion factor, such as the conversion from cups to gallons, is commonly used but cannot be remembered, other unit multipliers can be used. In the example below, the conversions 1 pint = 2 cups, 1 quart = 2 pints, and 1 gallon = 4 quarts are used in place of the conversion from cups to gallons.

Example 3: A fish tank can hold 40 cups of water. What is the capacity of the fish tank in gallons?

$$40\ \cancel{c} \bullet \frac{1\ \cancel{pt}}{2\ \cancel{c}} \bullet \frac{1\ \cancel{qt}}{2\ \cancel{pt}} \bullet \frac{1\ gal}{4\ \cancel{qt}} = \frac{40\ gal}{2 \bullet 2 \bullet 4} = 2.5\ gal$$

The fish tank can hold 2.5 gallons.

Note: When entering this in the calculator, parentheses must be used around the denominator because 40 is divided by the product of 2, 2, and 4.

CONVERTING BETWEEN SYSTEMS

Unit multipliers can be used to convert between systems of measurement, such as between the metric system and the US customary system. A chart of conversion factors between systems is shown below.

Length	Weight	Volume
1 in = 2.54 cm	1 oz ≈ 28.35 g	1 qt ≈ 0.95 L
1 mi ≈ 1.6 km	1 kg ≈ 2.2 lb	1 gal ≈ 3.8 L
1 m ≈ 1.1 yd	1 t ≈ 907 kg	Some of these conversions are rounded approximations, but they may be used as unit multipliers.

The following examples use unit multipliers from the table above to convert between the US customary system and the metric system.

Example 4: Carly is running a 10 km race for a charity event. How many miles will she run?

$$10\ \cancel{km} \bullet \frac{1\ mi}{1.6\ \cancel{km}} = \frac{10\ mi}{1.6} = 6.25\ mi$$

Carly will run 6.25 miles.

Example 5: Aaron lives in the US, and his cousin Bryan lives in the UK. They are discussing their weight-lifting goals. Bryan is using a 7.5-kg weight, and Aaron is using a 15-lb weight. Who is using a heavier weight?

$$7.5\ \cancel{kg} \bullet \frac{2.2\ lb}{1\ \cancel{kg}} = \frac{7.5 \bullet 2.2\ lb}{1} = 16.5\ lb$$

Bryan is using a heavier weight.

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Example 6: Find the number of liters in one pint of cream.

$$1 \text{ pt} \bullet \frac{1 \text{ qt}}{2 \text{ pt}} \bullet \frac{0.95 \text{ L}}{1 \text{ qt}} = \frac{0.95 \text{ L}}{2} = 0.475 \text{ L}$$

There are 0.475 liters in one pint.

CONVERTING AREA

Area is given in square units, such as square centimeters or square feet. When converting squared units, use two of the same unit multiplier. This will allow the squared units to cancel and will result in a new squared unit. It can help to write out squared units like ft^2 as ft • ft so the cancellation can be seen easily.

Example 7: Sierra is sewing aprons as presents for her cousins. She has a total of 18 yd^2 of fabric. How many square feet of fabric does Sierra have?

Multiply by the unit multiplier twice to cancel both units of yards.

$$18 \text{ yd} \bullet \text{yd} \bullet \frac{3 \text{ ft}}{1 \text{ yd}} \bullet \frac{3 \text{ ft}}{1 \text{ yd}} = \frac{18 \bullet 3 \bullet 3 \text{ ft} \bullet \text{ft}}{1} = 162 \text{ ft}^2$$

Sierra has 162 square feet of fabric.

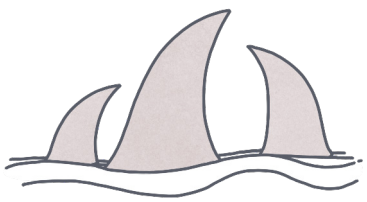
Example 8: Beth is designing a banner for her soccer team. The banner she wants to purchase is 20 ft^2. She has a piece of felt that is 8100 cm^2. Will Beth's piece of felt fit on the banner she wants to purchase?

To determine if the felt will fit, convert 20 ft^2 to cm^2.

$$20 \text{ ft} \bullet \text{ft} \bullet \frac{12 \text{ in}}{1 \text{ ft}} \bullet \frac{12 \text{ in}}{1 \text{ ft}} \bullet \frac{2.54 \text{ cm}}{1 \text{ in}} \bullet \frac{2.54 \text{ cm}}{1 \text{ in}}$$

$$= \frac{20 \bullet 12 \bullet 12 \bullet 2.54 \bullet 2.54 \text{ cm} \bullet \text{cm}}{1} = 18{,}580.608 \text{ cm}^2$$

The area of the banner is larger than the area of the felt, so the felt will fit on the banner.



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PRACTICE

A calculator may be used for this entire practice section.

1. Write two unit multipliers for each conversion factor.

a. 4 qt = 1 gal

_____ _____

b. 1.6 km = 1 mi

_____ _____

c. 1 oz = 28.35 g

_____ _____

d. 1 mi = 1760 yd

_____ _____

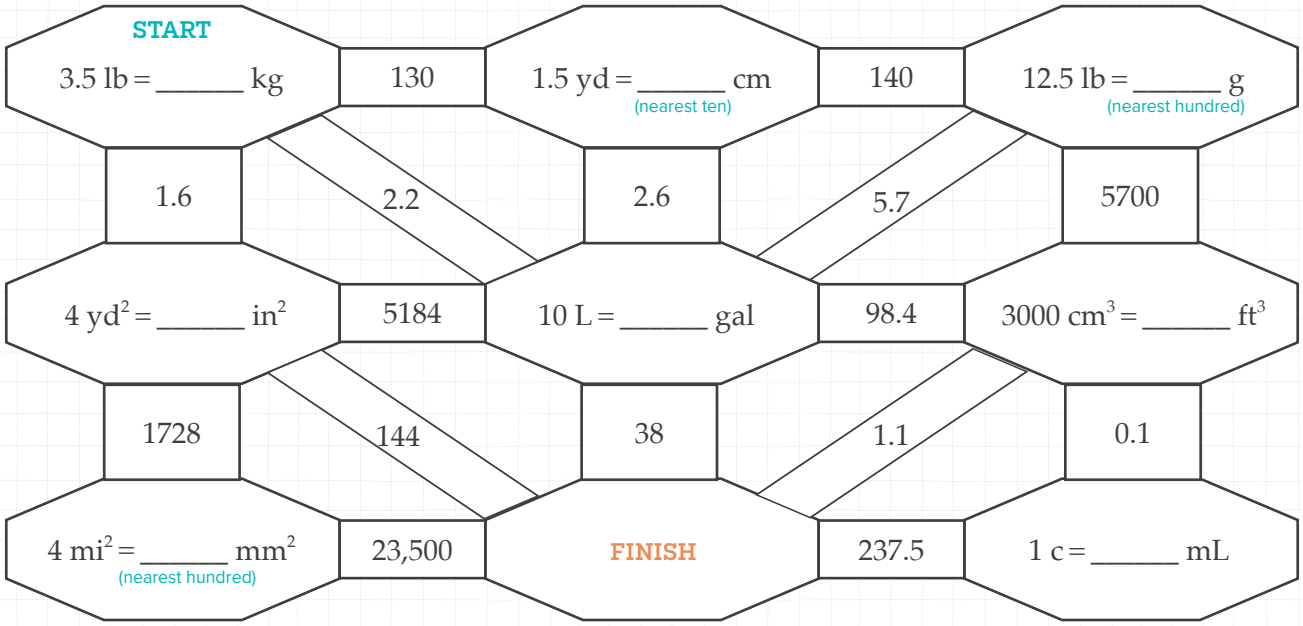
2. Beatrice has a recipe that calls for 1.5 quarts of milk, but she lives in a country where the milk is measured in liters. How many liters of milk will she need for the recipe?

_____

3. A soccer field is 7140 m^2 in area, and an American football field is 57,600 ft^2 in area. Determine which field is larger by converting 7140 m^2 to ft^2.

The _____ field is larger.

4. Begin at START. Complete each conversion to reach FINISH. Round to the nearest tenth unless otherwise indicated.



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Polygons and Interior Angles

WARM-UP

Two angles of a triangle have measures of 48° and 71°. Find the third angle measure.

_____

LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Practice section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

App Video

VIDEO PRACTICE

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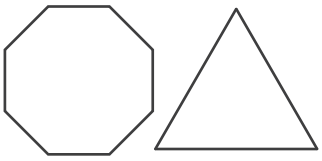
LESSON OVERVIEW

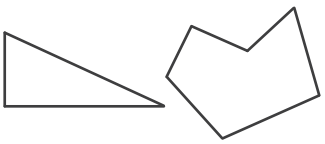
Shapes like triangles, squares, and pentagons are called polygons. Polygons can be found everywhere. Some naturally occur in nature, like hexagons in beehives, while others can be found in modern art or architecture.

Polygons are two-dimensional, closed shapes with straight sides. A ***regular polygon*** is a polygon that has all sides of equal length and all angles of equal measure. Some regular polygons have special names. An equilateral triangle is a regular triangle. A square is a regular quadrilateral.

Regular Polygons:



Irregular Polygons:



Not Polygons:



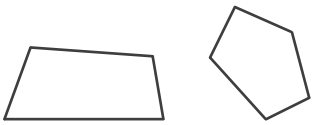
*curved sides *not closed

Polygons, whether regular or irregular, are named for their number of sides. Any polygon with more than 12 sides is referred to as an *n*-gon, where *n* is the number of sides.

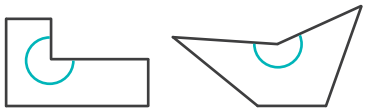
3 sides	4 sides	5 sides	6 sides	7 sides
triangle	quadrilateral	pentagon	hexagon	heptagon
8 sides	9 sides	10 sides	11 sides	12 sides
octagon	nonagon	decagon	hendecagon	dodecagon

Polygons can be convex or concave. In a ***convex polygon***, all interior angles measure less than 180°. In a ***concave polygon***, one or more interior angles measure more than 180°.

Convex Polygons:



Concave Polygons:



Note: The interior angle marked is greater than 180°.

Notice that convex polygons have all angles pointing outward from the middle of the shape. Concave polygons have one or more angles pointing inward toward the middle of the shape.

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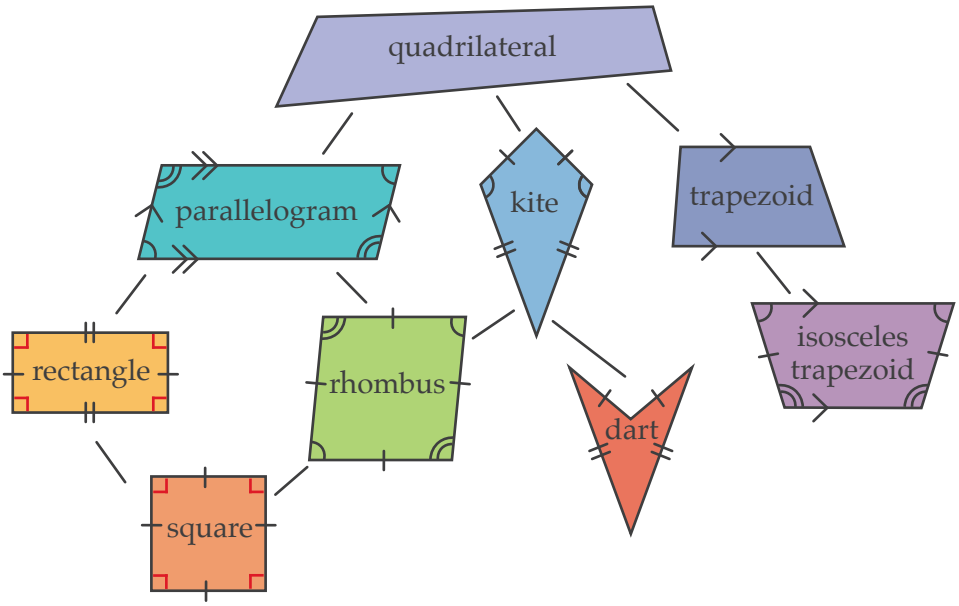
CLASSIFYING QUADRILATERALS

Quadrilaterals are polygons with exactly four sides. Below is a diagram showing the classifications of quadrilaterals in relation to one another. Shapes below other shapes are part of that classification. For example, a rectangle is a type of parallelogram, which is a type of quadrilateral. The definition of each shape is below the diagram. Note: The same number of arrows indicates parallel lines, and the same number of tick marks or arcs indicates congruent sides or angles.



Shape	Definition
parallelogram	a quadrilateral with two pairs of parallel sides
rectangle	a quadrilateral with four right angles
rhombus	a quadrilateral with four sides of equal length
kite	a quadrilateral with two sets of adjacent, congruent sides
square	a quadrilateral with four right angles and four sides of equal length
dart	a kite that has a concave angle
trapezoid	a quadrilateral with exactly one pair of parallel sides
isosceles trapezoid	a trapezoid in which the two non-parallel sides are the same length

Example 1: Classify a square in as many ways as possible and give a reason for each classification.

A square is a type of rectangle because it has four right angles.

A square is a type of rhombus because it has four sides of equal length.

A square is a type of parallelogram because it has two pairs of parallel sides.

A square is a type of kite because it has two sets of adjacent, congruent sides.

A square is a type of quadrilateral because it is a polygon with four sides.

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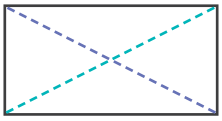
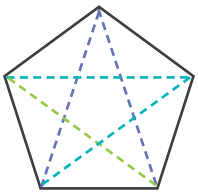
POLYGON ANGLES AND DIAGONALS

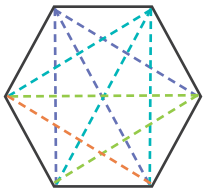
A ***diagonal*** is a straight line within a polygon that connects two of the polygon's nonadjacent vertices. The more sides and vertices a polygon has, the more diagonals it has. Note: To make it easier to see the number of diagonals in the polygons below, diagonals coming from the same vertex are the same color, starting at the top and going clockwise.



2 diagonals

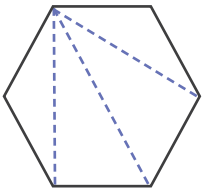


5 diagonals



9 diagonals

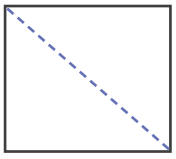
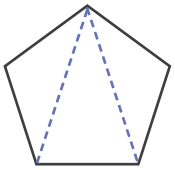
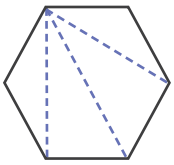
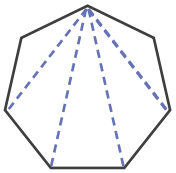
Some of the diagonals in a polygon can be used to divide the shape into triangles. When divided into the fewest number of triangles, the sum of the interior angles of the polygon can be found by using the fact that the sum of the interior angles of a triangle is 180°. Convex polygons can be divided into the fewest number of triangles by drawing all diagonals from one vertex as shown below.



The fewest number of triangles a hexagon can be divided into is four. The sum of the interior angles of a hexagon can be found by multiplying the number of triangles by 180°.

$4 \bullet 180° = 720°$ The interior angle sum of a hexagon is 720°.

The fewest number of triangles a polygon can be divided into is two less than the number of sides in the polygon. The table below shows the interior angle sum of polygons with n sides.

	Quadrilateral	Pentagon	Hexagon	Heptagon
Number of sides (n)	4	5	6	7
Number of triangles ($n-2$)	2 	3 	4 	5 
Interior angle sum $(n-2)\bullet 180°$	$2 \bullet 180° = 360°$	$3 \bullet 180° = 540°$	$4 \bullet 180° = 720°$	$5 \bullet 180° = 900°$

The fewest number of triangles a polygon can be divided into is $n-2$. The interior angle sum of a polygon is $(n-2)\bullet 180°$, where n is the number of sides of the polygon. To find a missing angle measure in a polygon, first calculate the interior angle sum. Then write and solve an equation with the sum of all angle measures equal to the interior angle sum.

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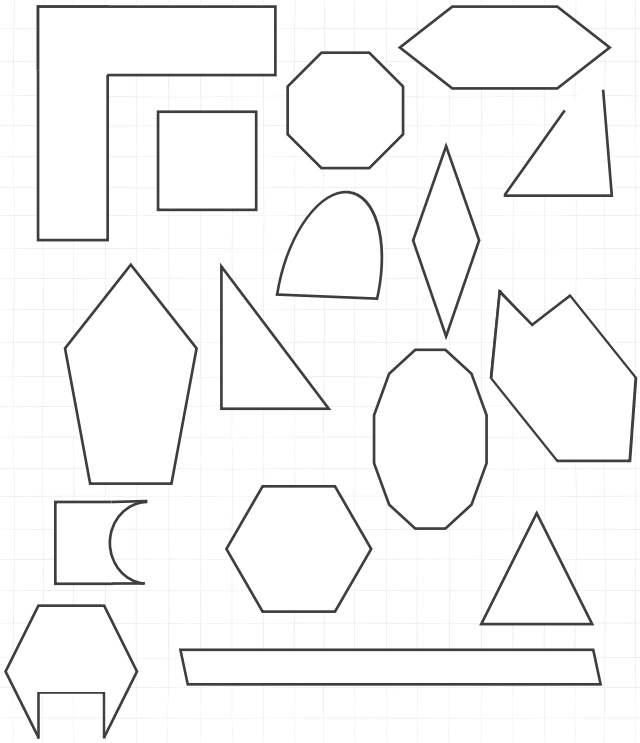
PRACTICE

1. Use the shapes below for Parts A–D.



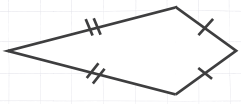
a. Cross off any shape that is not a polygon.

b. On each polygon, write the name according to the number of sides. For example, write "quadrilateral" instead of "rectangle" or "square."

c. Circle any regular polygons.

d. Color in any concave polygons.

2. Classify the polygons in as many ways as possible and give a reason for each classification.

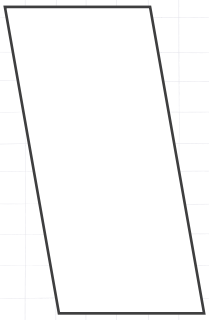
a. 

Shape	Reason

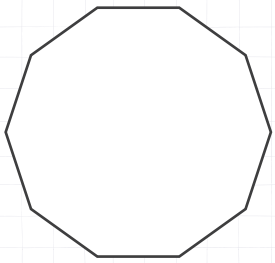
b. 

Shape	Reason

3. Draw diagonals to divide each shape into the fewest number of triangles. Then determine the sum of the interior angles of the polygon.

a. 

Interior angle sum: _____

b. 

Interior angle sum: _____

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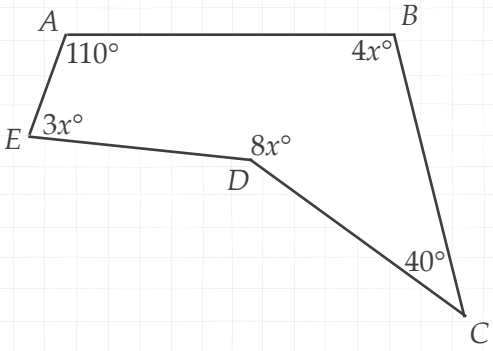
4. Find the indicated angle measures in each polygon.

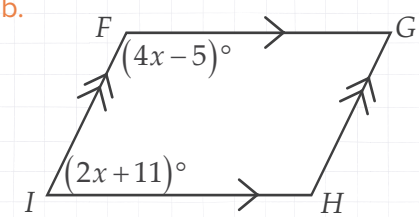
✦ Hint: First use the interior angle sum formula. Then write an equation setting the sum of all angles equal to the interior angle sum.

a.



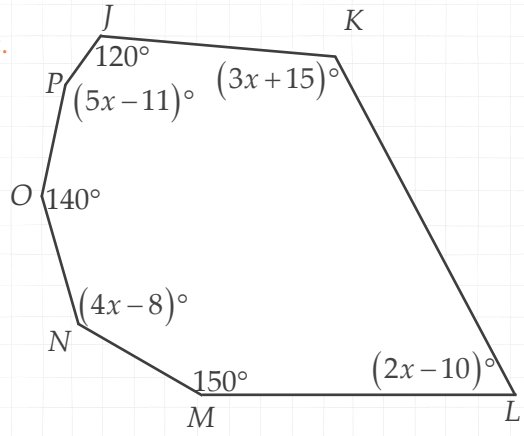
$m\angle B =$ _____ $m\angle D =$ _____

b.

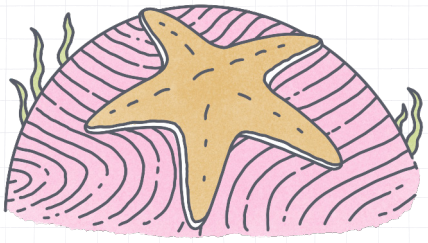


$m\angle G =$ _____ $m\angle H =$ _____

c.

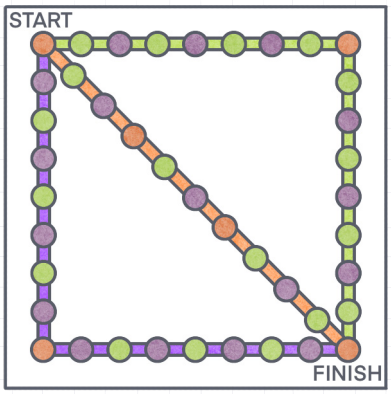


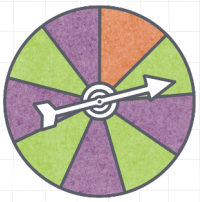
$m\angle K =$ _____ $m\angle N =$ _____



REVIEW

1. Noah is creating a board game. The board on which the game is played is a 15-inch by 15-inch square. A diagram of the game board and game spinner are shown below. Use the given information to answer the questions.





a. At the beginning of the game, each player will spin the spinner to determine the color of the path he or she will follow from START to FINISH. Write the probability of spinning each path as a fraction. L63

orange: _____ green: _____ purple: _____

b. What is the length of the orange path (diagonal distance of the game board)? Round the answer to the nearest tenth of an inch. L53

✦ Hint: Use the given game board dimensions and the Pythagorean theorem.

_____

c. What is the probability of spinning orange and then green? Round to the nearest hundredth of a percent. L64

_____

d. Classify the triangle formed by the green path and the orange path by angles and by sides. L72

_____ and _____

e. Noah is designing a triangular image to be printed on the board. He is considering making the sides of the triangle 2 inches, 1.5 inches, and 4 inches. Determine if a triangle can be formed by the lengths Noah is considering. Write "yes" or "no" on the line. L72

_____

f. Several families from Noah's church have agreed to test his game. If there are 35 people who will test the game, and the ratio of children to adults is 3 : 2, how many adults will test the game? L66

✦ Hint: Find the ratio of adults to total people, and then create a proportion.

_____ adults

g. Once the game is produced, Noah estimates that 32% of total game sales (revenue) will be profit. If Noah sets a goal to make $5000 in profit in the first six months, how much does he need in sales revenue? L57

_____

h. If each game is sold for the same unit price, the sales revenue can be modeled by the equation $y = kx$, where y is revenue, k is the price of each game, and x is the number of games sold. If each game is sold for $25, use the equation and the answer to Part G to determine the number of games Noah needs to sell in the first six months to meet his goal. L44

_____ games

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Logic Lesson 3



Songkran is a festival commemorating the original Thai New Year. Water is used in the celebrations as a symbol of purification and renewal and as an element of fun. Songkran is sometimes referred to as the Water Festival, and friendly water fights are held in the streets. A few Songkran traditions are detailed in the puzzles contained in this logic lesson. This lesson has no video or review problems.

1. Suppose an extended family takes a family photo during a celebration. The photo consists of one grandfather, one grandmother, two fathers, two mothers, six children, four grandchildren, two brothers, two sisters, three sons, three daughters, one father-in-law, one mother-in-law, and one daughter-in-law. What is the fewest number of people who could have been in the photo?

_____ people

2. Songkran is typically held for three days each year: April 13th, 14th, and 15th. Homes and public and religious buildings are deep cleaned in preparation for the festival and as a symbol of a fresh start for the new year.

Create the numbers 13, 14, and 15 using exactly five 5's and mathematical operations. Parentheses may be used, and 5's may be put together to make a larger number (like 55). An example of using five 5's to create the number 12 is shown below.

$$12 = 5 + 5 + \frac{5+5}{5}$$

a. $13 =$ _____

b. $14 =$ _____

c. $15 =$ _____

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5. Some additional Songkran traditions include building sand pagodas outside temples, dressing in traditional Thai clothing to perform traditional dances, releasing fish or birds into the wild, making traditional food to share with family and/or religious leaders, making floral garlands to give to elder relatives, and playing games. Suppose that on the day after Songkran, four Thai children were asked their favorite day and event of the recent festival. Each child shared a different favorite event, but because the festival lasts only three days, two of the children stated that their favorite day of the festival was the second day, April 14th. Read the clues to find each child's favorite event and day.

✦ Hints: Once you know something for certain, put a ✓ in that square and fill in the rest of the row and column of that 4 x 4 box with Xs. You may need to go through the clues more than once.

Clue #1: A boy and a girl each stated that April 14th was their favorite day of the festival, but neither of them stated that the Water Fight or Dancing was their favorite event.

Clue #2: The boy who chose the Water Fight as his favorite enjoyed the final day of the festival the most.

Clue #3: The boy who chose Making Garlands enjoyed the second day of the festival the most.

Clue #4: Dara's favorite day was not the first day.

Clue #5: Phet did not choose Making Garlands.

	Favorite Day				Favorite Event			
	April 13th	April 14th	April 14th	April 15th	Dancing	Making Garlands	Water Fight	Building Sand Pagodas
Dara (girl)								
Mali (girl)								
Saksit (boy)								
Phet (boy)								
Dancing								
Making Garlands								
Water Fight								
Building Sand Pagodas								



a. Dara's favorite day: _____
Dara's favorite event: _____

b. Mali's favorite day: _____
Mali's favorite event: _____

c. Saksit's favorite day: _____
Saksit's favorite event: _____

d. Phet's favorite day: _____
Phet's favorite event: _____

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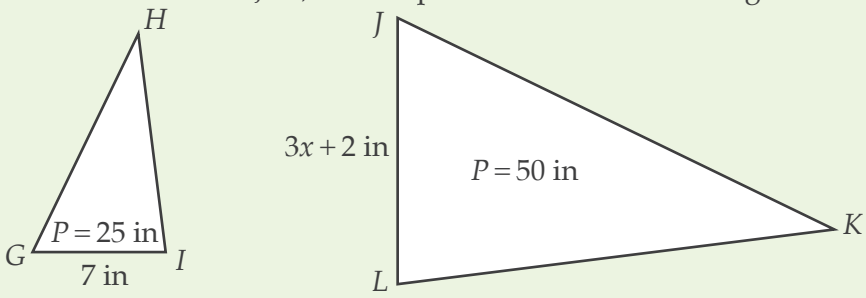
Drawings and Constructions

SUPPLIES: ruler, compass, and protractor

WARM-UP

Given that $\triangle GHI \sim \triangle JKL$, use the perimeters to find the length of side JL.



_____

LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Practice section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

App Video

VIDEO PRACTICE

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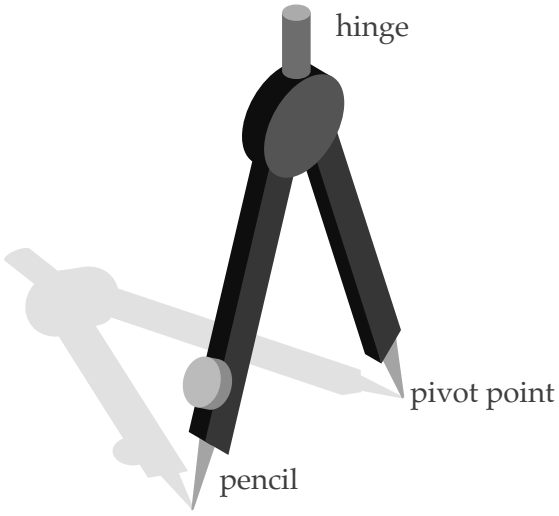
LESSON OVERVIEW

Geometric constructions are different from freehand drawings because of the precision used when completing them. Constructions only use two tools: a straightedge and a compass. A protractor is not a valid tool for creating constructions. Drawings can be made freehand or with other tools like a protractor or ruler.

USING A COMPASS

A compass is a tool used to construct circles and arcs, which can be used to construct many other figures. A compass can be held at the top as if it were a key being turned on the table. The compass contains a hinge for adjusting the distance between the pivot point and the pencil that are at the ends of the compass. The pivot point marks the center of a circle that would be created if the compass were turned a complete 360°. The distance between the pencil and pivot point is the length of the radius of that circle.



CONSTRUCTING PERPENDICULAR BISECTORS

A ***perpendicular bisector*** is a line, ray, or line segment that divides a line segment into two congruent parts and is perpendicular to the line segment.

Steps for constructing a perpendicular bisector for line segment *AB*:

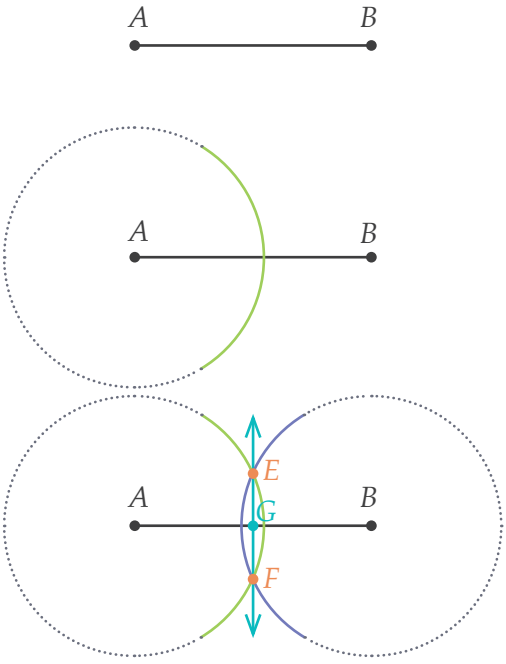
1. Place the pivot point of the compass on one endpoint of the line segment. Open the compass to more than half the length of the line segment.

2. Draw part of a circle (an arc). This arc should extend above and below the line segment.

3. Keep the same radius on the compass and move the pivot point to the other endpoint of the line segment. Draw another arc that intersects the first arc in two places.

4. Draw points at the intersections of the arcs.

5. Use a straightedge to connect the intersection points. Line *EF* is a perpendicular bisector for line segment *AB*.



Note: The perpendicular bisector creates two congruent line segments. If point *G* is placed at the intersection of line segment *AB* and the perpendicular bisector, line segment *AG* is congruent to line segment *GB*.

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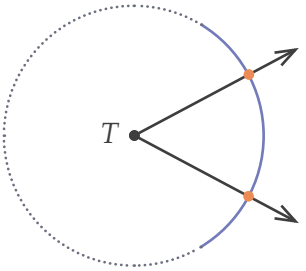
CONSTRUCTING ANGLE BISECTORS

An ***angle bisector*** is a line, ray, or line segment that divides an angle into two angles of equal measure.



Steps for constructing an angle bisector for angle *T*:

1. Place the pivot point of the compass on the vertex.

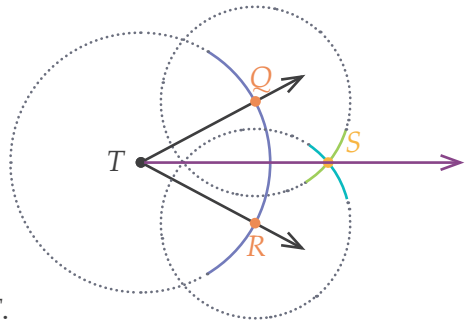
2. Draw part of a circle (an arc) that intersects both sides of the angle.

3. Draw points at the intersections of the arc and the angle.

4. Place the pivot point of the compass on one of the intersection points. Open the compass to more than half the distance to the other intersection point. Draw an arc.

5. Keep the same radius on the compass and move the pivot point to the other point of intersection on the angle. Draw another arc that intersects the arc from Step 4.

6. Draw a point at the intersection of the arcs from Steps 4 and 5. Use a straightedge to connect this point with the vertex of the angle. Ray *TS* is the angle bisector for angle *T*.



Note: The angle bisector creates two congruent angles. If the intersection points from Step 3 are named *Q* and *R* as shown in the figure, then angle *QTS* is congruent to angle *STR*.

Perpendicular bisectors and angle bisectors can be constructed using only a compass and straightedge. There are many other types of geometric constructions that are taught in higher-level geometry courses. The following instructions use a ruler and a protractor, so they are considered drawings, not constructions.

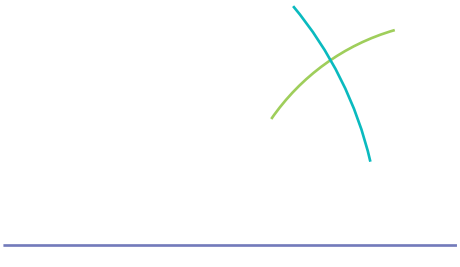
DRAWING A TRIANGLE GIVEN THREE SIDE LENGTHS

To draw a triangle given three side lengths, a compass and ruler can be used.

Steps for drawing a triangle with side lengths of 3 cm, 5 cm, and 6 cm:

1. Use the ruler to measure and draw the first side. 6 cm is shown first here, but any side can be drawn first.

2. Set the radius of the compass to the length of another side. 3 cm is used next here. Put the pivot point of the compass on one endpoint of the first side and draw an arc above that side. Any point on this arc is 3 cm from that endpoint.



Note: Some compasses have measurements on the hinge for setting the radius. A ruler can also be used to measure the distance between the pivot point and pencil.

3. Set the radius of the compass to the length of the third side, which is 5 cm in this example. Put the pivot point on the other endpoint of the first side and draw another arc that intersects the first arc. Any point on this arc is 5 cm from that endpoint.

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PRACTICE

1. a. Construct a perpendicular bisector for the line segment below.

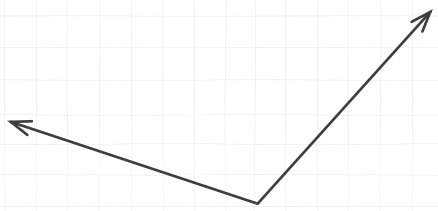
b. Verify that the construction in Part A bisects the line segment by measuring each side of the line segment and writing the measurements below.

Left side: _____ Right side: _____

c. Verify that the construction in Part A is perpendicular to the line segment by measuring the angle made with the given line segment.

_____

2. a. Construct an angle bisector for the angle below.



b. Verify that the construction in Part A bisects the angle by measuring the two angles created by the bisector.

Left angle measure: _____

Right angle measure: _____

3. Draw a triangle for each set of side lengths.

a. 2 in, 2.5 in, 3 in

b. 4 cm, 3 cm, 4 cm

4. Construct a triangle for each set of angle measures.

a. 55°, 70°, 55°

b. 115°, 30°, 35°

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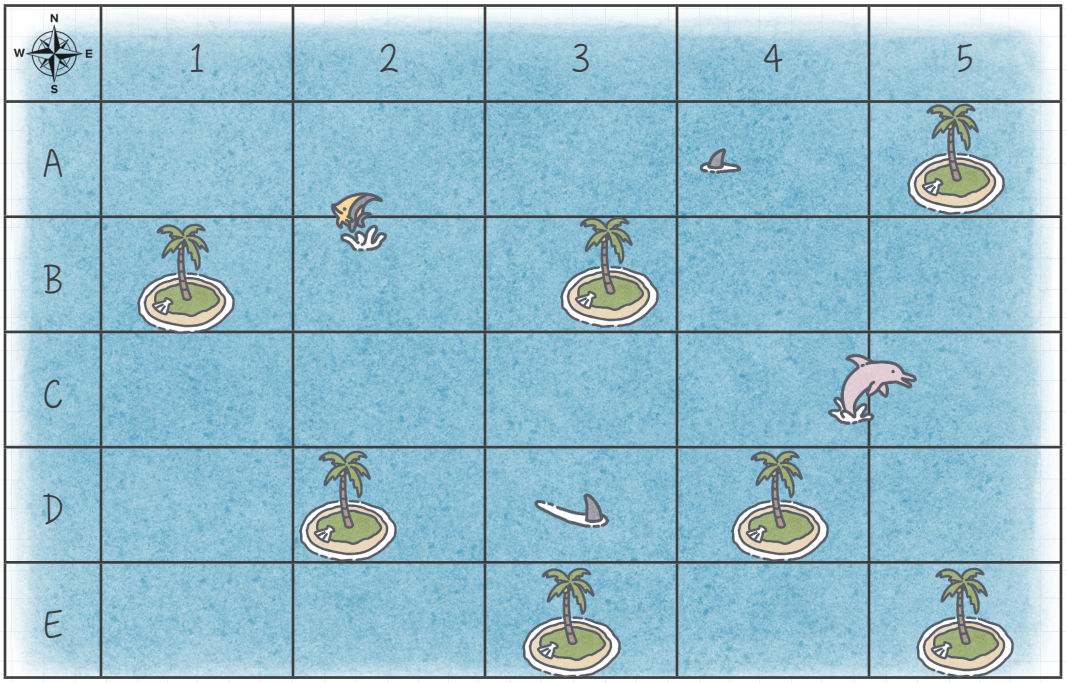
5. A treasure map is shown below. Follow the clues to find the location of the treasure.



a. Draw a line segment from the base of the palm tree in D2 to the base of the palm tree in B1.
✦ Hint: The base of the palm tree is where the tree meets the island.

b. Construct a perpendicular bisector for the line segment drawn in Part A.

c. The perpendicular bisector created in Part B forms four angles where it intersects the line segment from Part A. Using only a compass and straightedge, bisect the angle that faces east.

d. Extend the angle bisector from Part C until it hits an island. The treasure is on this island. In what section of the map is the treasure island located?

_____



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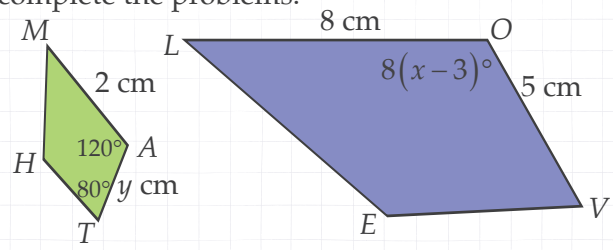
REVIEW

A calculator may be used for this entire review section.

1. Given that $LOVE \sim MATH$, use the figures to complete the problems.



a. Name the polygons above according to their number of sides. L73

_____

b. Circle the correct word from each row to describe the polygons. L73

Circle one: regular / irregular

Circle one: concave / convex

c. Determine the value of x. L76

_____

d. Determine the value of y. L76

_____

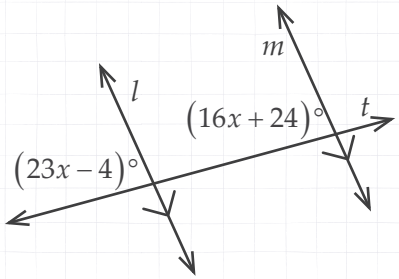
2. Convert 23,225 cm^2 to square feet. Round to the nearest square foot. L68

_____

3. Oliver builds wooden scale models of farm equipment. The diameter of each tractor wheel on a model is 3 inches, and the diameter of the actual tractor wheel is 42 inches. What scale factor did Oliver use for the model tractor? L69

_____

4. Use the figure below to answer the questions.



a. What is the relationship between the two angles whose measures are labeled with expressions? L71

_____ angles

b. Are angles with this relationship congruent or supplementary? Circle one. L71

congruent supplementary

c. Use the answer from Part B and the given expressions to write and solve an equation to find the value of x and the measures of the angles. L25, L31

$x =$ _____ $(23x - 4)° =$ _____

$(16x + 24)° =$ _____

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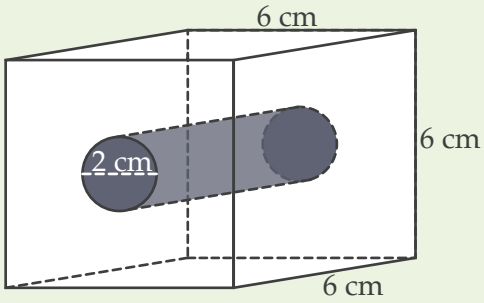
Solving One-Step and Two-Step Inequalities

WARM-UP

Find the volume of the composite solid.



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LESSON

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App Video

VIDEO PRACTICE

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LESSON OVERVIEW

An ***inequality*** is a comparison of two expressions that does not use an equal sign. Inequality symbols and their meanings are shown in the chart below.

Symbol	Meaning
$<$	less than
$>$	greater than
$\leq$	less than or equal to
$\geq$	greater than or equal to
$\neq$	not equal to

$x < 7$ means that the value of x must be less than 7. Any number less than 7 is a possible value for x. This includes numbers like 6.98, 0, and –46. Any number that is *not* less than 7 is not a possible value for x. Some numbers that do not satisfy the inequality include 7, 45, and 1258.

ONE-STEP INEQUALITIES

An inequality is solved similarly to how an equation is solved. Always perform the same operations on both sides of the inequality to maintain the same relationship that was originally expressed.

Example 1: Solve for a.

$$\begin{aligned} a + 11 &> -19 \\ a + 11 - 11 &> -19 - 11 \\ a &> -30 \end{aligned}$$

Example 2: Solve for x.

$$\begin{aligned} x - 6 &< 18 \\ x - 6 + 6 &< 18 + 6 \\ x &< 24 \end{aligned}$$

To determine if a number satisfies an inequality, substitute the number into the inequality and see if a true statement results. If the statement is true, the number satisfies the inequality. If the statement is false, the number does not satisfy the inequality.

Example 3: Determine if the numbers in the set $\{0,\ 2.5,\ 11,\ -11,\ -12,\ -23\}$ are solutions to the inequality $b - 12 \geq -23$.

Check each value in the set in the original inequality.

$0 - 12 = -12$	$-12 \geq -23$	0 satisfies the inequality.
$2.5 - 12 = -9.5$	$-9.5 \geq -23$	2.5 satisfies the inequality.
$11 - 12 = -1$	$-1 \geq -23$	11 satisfies the inequality.
$-11 - 12 = -23$	$-23 \geq -23$	–11 satisfies the inequality.
$-12 - 12 = -24$	$-24 < -23$	–12 does *not* satisfy the inequality.
$-23 - 12 = -35$	$-35 < -23$	–23 does *not* satisfy the inequality.

The numbers 0, 2.5, 11, and –11 are all solutions to the inequality $b - 12 \geq -23$. The numbers –12 and –23 are not solutions.

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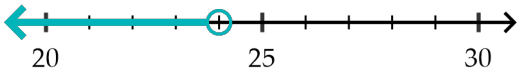
Inequalities have an infinite number of solutions. Solutions can be written with an inequality symbol, and they can also be graphed on a number line. Below are the graphs of the solutions to Examples 1, 2, and 3. Any number in the shaded part (including decimal values) is a solution to the inequality. An open circle on the number is used when the symbol is < or >. The number at the circle is not part of the solution set. A closed circle on the number is used when the symbol is ≤ or ≥. The number at the circle is part of the solution set.

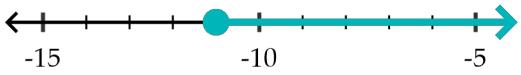
$a > -30$



$x < 24$



$b \geq -11$

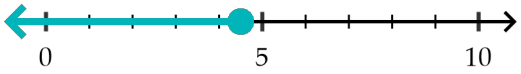


Example 4: Solve and graph the inequality.

$$5q \leq 22.5$$

$$\frac{5q}{5} \leq \frac{22.5}{5}$$

$$q \leq 4.5$$



When multiplying or dividing both sides of an inequality by a negative value, the inequality sign switches directions. The table below demonstrates how the relationship between numbers changes when both sides of an inequality are multiplied or divided by a negative number.

Original Inequality	Multiply by 2	Multiply by -2	Divide by 6	Divide by -6
$18 > 12$	$18 \cdot 2 \quad 12 \cdot 2$ $36 > 24$	$18 \cdot (-2) \quad 12 \cdot (-2)$ $-36 < -24$ sign changes	$18 \div 6 \quad 12 \div 6$ $3 > 2$	$18 \div (-6) \quad 12 \div (-6)$ $-3 < -2$ sign changes

Anytime both sides of an inequality are multiplied or divided by a negative number, remember to switch the direction of the inequality symbol.

Example 5: Solve and graph the inequality.

$$-8f \geq 48$$

$$\frac{-8f}{-8} \geq \frac{48}{-8}$$

$$f \leq -6$$

Note: The sign switches.



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TWO-STEP INEQUALITIES

Inequalities may require multiple steps to solve. As when solving equations, isolate the variable by performing the order of operations in reverse. First undo any addition or subtraction. Then undo any multiplication or division. Remember to switch the inequality symbol when multiplying or dividing by a negative number.

Example 6: Solve and graph the inequality.

$12t + 15 > 39$ — Subtract 15 from both sides.

$12t + 15 - 15 > 39 - 15$

$12t > 24$ — Divide both sides by 12.

$\frac{12t}{12} > \frac{24}{12}$

$t > 2$



Note: There is an open circle on 2 because 2 is not part of the solution set.

Example 7: Solve and graph the inequality.

$-\frac{3}{4}p + 21 \geq 24$ — Subtract 21 from both sides.

$-\frac{3}{4}p + 21 - 21 \geq 24 - 21$

$-\frac{3}{4}p \geq 3$ — Multiply both sides by the reciprocal of $-\frac{3}{4}$.

$-\frac{4}{3} \cdot \left(-\frac{3}{4}p\right) \geq 3 \cdot \left(-\frac{4}{3}\right)$ — Multiplying by a negative switches the inequality symbol.

$p \leq -4$



Example 8: Solve and graph the inequality.

$-3x - 15 \leq 6$ — Add 15 to both sides.

$-3x - 15 + 15 \leq 6 + 15$

$-3x \leq 21$ — Divide both sides by –3.

$\frac{-3x}{-3} \leq \frac{21}{-3}$ — Dividing by a negative switches the inequality symbol.

$x \geq -7$



☆ KEY INFORMATION

Use an open circle for < or >. Use a closed circle for ≤ or ≥.

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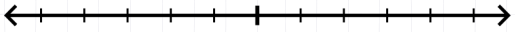
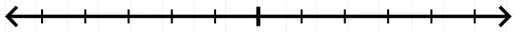
PRACTICE

Solve and graph each inequality. Find the solution graph in the table on the next page, and cross off the word or phrase next to it. Once all problems have been completed, write the remaining words or phrases (from top to bottom) on the lines at the bottom of the page to discover the answer to the joke.

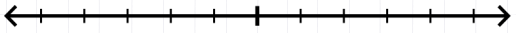
1. $a + 2 < 7$ _____



2. $b - 4 > 3$ _____



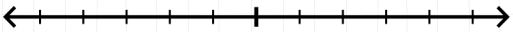
3. $3c \geq -27$ _____



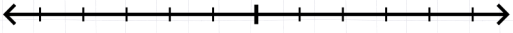
4. $-8d < 20$ _____



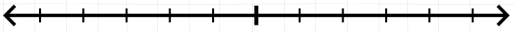
5. $2e + 5 \leq 17$ _____



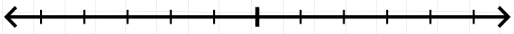
6. $4f - 6 > -30$ _____



7. $-g + 2 \geq 17$ _____



8. $\frac{1}{2}h - 4 \leq 7$ _____



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Unit 3 Review

SUPPLIES: compass, protractor, ruler

Complete this Unit Review to prepare for the Unit Assessment. There is no video, lesson, or practice. Because Unit Reviews include practice for an entire unit, they may take longer than regular lessons, and students may decide to take two days to finish.

Tatiana is beginning a volunteer position for a marine conservation organization. Complete these problems related to her work. Round any decimals to the nearest hundredth or hundredth of a percent.

Calculators may be used when needed for this entire review.

LESSONS 61–62

1. a. Last year, the organization had 15 seasonal volunteers. It hopes to increase its volunteer base by 40% this year. How many volunteers does it hope to have this year?

_____

b. The organization has already raised $8280 this year. They had raised $7200 at this time last year. What percent increase is this?

_____

c. At the first beach cleanup event, the group picked up 78.2 kg of trash, which is a 15% decrease from last year. How much trash did they collect at this beach last year?

_____

2. a. Tatiana wants to give some of her savings to support the organization. How much money is in her account if it began with a principal of $400 and earned 4.5% simple interest annually for 10 years?

_____

b. The organization has an endowment that is worth $2,200,000 after earning 7% interest compounded annually for the last 30 years. How much was the initial endowment (principal)? Round to the nearest thousand.

_____

LESSONS 63–64

3. The organization works with 175 scientists. Of these scientists, 56 are biologists, 62 are ecologists, and 45 are oceanographers.

a. What is the probability that a randomly chosen scientist in the organization is either a biologist or an ecologist?

_____

b. How many different three-person teams are possible with exactly one biologist, one ecologist, and one oceanographer?

_____

c. If a team of two scientists were created, what is the probability that no one on the team is a biologist, ecologist, or oceanographer?

Hint: These are dependent events.

_____

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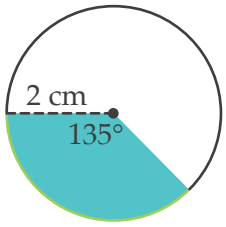
LESSONS 79–80

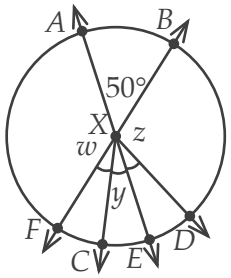
17. Tatiana is in charge of designing buttons for an awareness campaign and is considering different designs.

a. If the radius of the button is 2 cm, determine the green arc length and the area of the blue sector below.



Green arc length: _____ Blue sector area: _____

b. Find the missing angle measures.



$w =$ _____

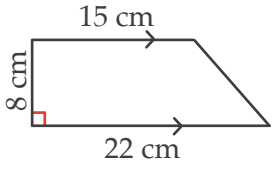
$y =$ _____

$z =$ _____

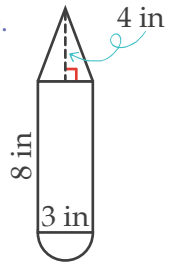
LESSONS 80–81

18. The conservatory is planning on repainting some of its walls and displays. Determine the area of the following figures.

a.



b.

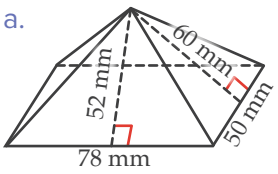


_____ _____

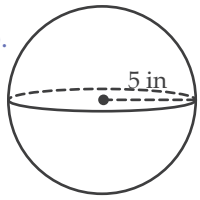
LESSONS 82–83

19. Determine the surface area of the following solids.

a.



b.

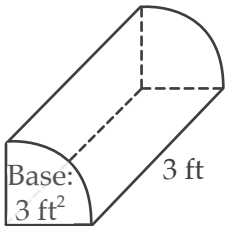


_____ _____

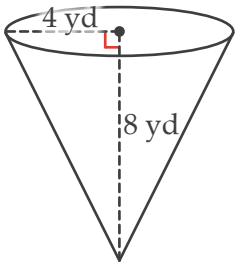
LESSONS 84–86

20. The marine conservation organization also works with aquariums. Find the volume of the following tanks at a nearby aquarium.

a.



b.



_____ _____

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Unit 3 Assessment

○ This assessment covers concepts taught in Unit 3. Problems are designed to assess multiple skills. Read the instructions carefully and do not rush through the problems.

○ You may use the Reference Chart for the assessment. Lesson numbers are given by each problem so you can review lessons for any answers that are incorrect.


Calculators may be used throughout this assessment.

1. The value of Cynthia's house has increased from $350,000 to $420,000. Find the percent increase on the value of her home. L61

_____

2. A savings account starts with $450. Find the new balance in the account after 12 years of 3.5% interest compounded annually. L62

_____

3. A jar contains 120 pink beads, 80 yellow beads, 100 blue beads, 150 green beads, and 50 white beads. L63

   a. Find the probability of randomly picking a bead of each color from the jar on one pick.

   Pink: _____ Green: _____

   Yellow: _____ White: _____

   Blue: _____

   b. Find the expected number of beads of each color that would likely be in a scoop of 50 beads.

   Pink: _____ Yellow: _____

   Blue: _____ Green: _____

   White: _____

4. Jessie was given six pairs of earrings for her birthday. If there are two pairs of studs, one pair of hoops, and three dangly pairs, find the probability that she will take out a dangly pair first and a pair of hoops second. L64

_____

5. A package of 46 cookies costs $3.78. Find the unit rate of cost per cookie to the nearest cent. L65

_____

6. Rob's pasta recipe uses 500 grams of uncooked pasta and half of a yellow onion. If he wants to use 0.75 onions, how much pasta should he use? L66

_____

7. Perform the operations. L67

   a. 4 lb 6 oz • 3 = _____

   b. 2 ft 5 in + 4 ft 8 in = _____

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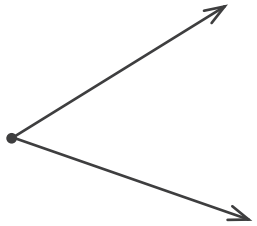
8. Siera is flying from California to Heathrow, London. Her luggage must be below 40 lbs on her departure flight. When she flies home, she must check her luggage on a scale that measures in kg. Use the conversion 1 kg ≈ 2.2 lb to determine the maximum number of kg her luggage can weigh. Round to the nearest hundredth. L68

_____

9. A redwood tree is 320 feet tall. In a picture, the tree measures 4 inches tall. What is the scale from the real tree to the picture? L69

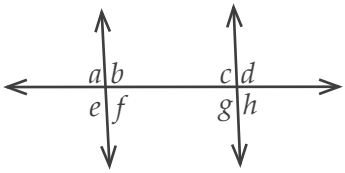
_____

10. Classify the angle by measure. L70



Circle one: acute obtuse right

11. List the vertical angle pairs from the diagram. L71



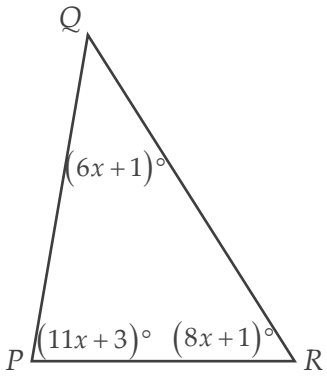
_____ and _____

_____ and _____

_____ and _____

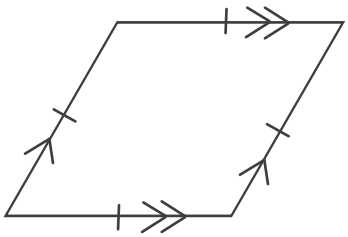
_____ and _____

12. Find the measure of angle P. L72



_____

13. Classify the polygon in as many ways as possible and give a reason for each classification. L73



Shape	Reason

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