UNITS
1–4

Simply Good and Beautiful

PRE-ALGEBRA

ANSWERS & SOLUTIONS

MATH
8

The Good and the Beautiful
CURRICULUM

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Place Value and Estimation

WARM-UP

a. 60 b. 24 c. 51

PRACTICE

1. The digit to the right of the hundreds place (7) is greater than 5, so round up.
   1400

2. $6.3\overline{81} = 6.3818181...$
   The digit to the right of the hundred thousandths place (8) is greater than 5, so round up.
   6.38182

3. 3

4. $(3 \bullet 100{,}000) + (2 \bullet 1000) + (4 \bullet 100)$
   $= 300{,}000 + 2000 + 400$
   $= 302{,}400$

5. 12,000.006

6. 405,213

7. 18,324 rounds to 18,000.
   $18{,}000 \div 9 = 2000$

8. Numbers with a whole number part of 1 are smallest. Compare the tenths place in these numbers: 1.23 1.203 1.03

   1.03 is the smallest. Then compare the hundredths place in the other two numbers. 1.203 is smaller than 1.23.

   2.31 is greater than numbers with a whole number part of 1 but less than numbers with a whole number part of 12.

   For numbers with a whole number part of 12, look at the tenths place. 12 12.3

   12 has a 0 in the tenths place, so it is smaller than 12.3.

   Least to greatest:

   1.03, 1.203, 1.23, 2.31, 12, 12.3

   Riddle answer: OWL-GEBRA

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REVIEW

More than one answer is possible for each problem. A possible solution is given.

1. $6 \bullet 4 - 2$
$= 24 - 2$
$= 22$

2. $\frac{7+5}{4}$
$= \frac{12}{4}$
$= 3$

3. $5 + 5$
$= 10$

4. $7 + 3 + 1$
$= 10 + 1$
$= 11$

5. $\frac{8 \bullet 2 - 6}{5}$
$= \frac{16 - 6}{5}$
$= \frac{10}{5}$
$= 2$

6. $\frac{8-1}{7} \bullet 7 \bullet 2$
$= \frac{7}{7} \bullet 7 \bullet 2$
$= 1 \bullet 7 \bullet 2$
$= 7 \bullet 2$
$= 14$

7. $8 \div 2 + 2$
$= 4 + 2$
$= 6$

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Decimals and Fractions

WARM-UP

a. $22 \bullet 4 = 88$

$21.843 \bullet 3.93 \approx 88$

b. $56 \div 8 = 7$

$55.5 \div 8.24 \approx 7$

c. $11 \bullet 5 = 55$

$11.058 \bullet 5.321 \approx 55$

PRACTICE

1. a. $0.65 = \frac{65}{100} = \frac{13}{20}$

b. $12.32 = 12\frac{32}{100} = 12\frac{8}{25}$

c. $10.98 = 10\frac{98}{100} = 10\frac{49}{50}$

d. $6.4 = 6\frac{4}{10} = 6\frac{2}{5}$

2. a.
$$\begin{array}{r} 0.6 \\ 5\overline{)3.0} \\ -\,30 \\ \hline 0 \end{array}$$
$\frac{3}{5} = 0.6$

b.
$$\begin{array}{r} 0.466\ldots \\ 15\overline{)7.000\ldots} \\ -\,60 \\ \hline 100 \\ -\,90 \\ \hline 100 \\ -\,90 \\ \hline 10 \end{array}$$
$\frac{7}{15} = 0.4\overline{6}$

3. a. same denominator, $5 > 2$

$\frac{5}{3} > \frac{2}{3}$

b. same numerator, $7 > 6$

$\frac{5}{7} < \frac{5}{6}$

c. Half of 9 is 4.5.

$\frac{5}{9} > \frac{1}{2}$

Half of 13 is 6.5.

$\frac{6}{13} < \frac{1}{2}$

$\frac{5}{9} > \frac{6}{13}$

d. Half of 22 is 11.

$\frac{11}{22} = \frac{1}{2}$

Half of 16 is 8.

$\frac{5}{16} < \frac{1}{2}$

$\frac{11}{22} > \frac{5}{16}$

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4. a. Half of 4 is 2.

$\frac{3}{4} > \frac{1}{2}$

Half of 5 is 2.5.

$\frac{2}{5} < \frac{1}{2}$

$\frac{4}{3} = 1\frac{1}{3}$

Greatest to least:

$\frac{4}{3}, \frac{3}{4}, \frac{1}{2}, \frac{2}{5}$

b. same numerator (smaller denominators mean larger fractions)

Greatest to least:

$\frac{7}{9}, \frac{7}{11}, \frac{7}{12}, \frac{7}{13}$

5.

$\frac{2}{5}$	A 1.375	B $2\frac{4}{9}$
C 0.476	D 1.45	$\frac{119}{250}$
$2.\overline{4}$	$1\frac{9}{20}$	$1\frac{3}{8}$
E $1\frac{3}{11}$	$1.\overline{27}$	0.4

Detailed work is shown below. Work may vary. Students may have converted decimal values to fractions or fractions to decimals to find equivalent numbers.

A $1.375 = 1\frac{375}{1000} = 1\frac{15}{40} = 1\frac{3}{8}$

B
$$\begin{array}{r} 0.44... \\ 9\overline{)4.00...} \\ -\ 36 \\ \hline 40 \\ -\ 36 \\ \hline 4 \end{array}$$

C $0.476 = \frac{476}{1000} = \frac{119}{250}$

D $1.45 = 1\frac{45}{100} = 1\frac{9}{20}$

E
$$\begin{array}{r} 0.2727... \\ 11\overline{)3.0000...} \\ -\ 22 \\ \hline 80 \\ -\ 77 \\ \hline 30 \\ -\ 22 \\ \hline 80 \\ -\ 77 \\ \hline 30 \end{array}$$

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REVIEW

1. $$\begin{array}{r} 83 \\ 13\overline{)1079} \\ \underline{-104} \\ 39 \\ \underline{-39} \\ 0 \end{array}$$

83 packages

2. a. Minutes practiced each week:

$$\begin{array}{r} {}^{2} \\ 45 \\ \underline{\times\ \ 5} \\ 225 \end{array}$$

225 minutes

Minutes practiced in a year:

$$\begin{array}{r} {}^{1}\,{}^{2}\,{}^{1} \\ 225 \\ \underline{\times\ \ 52} \\ 450 \\ \underline{+11250} \\ 11700 \end{array}$$

11,700 minutes

b. $$\begin{array}{r} 195 \\ 60\overline{)11700} \\ \underline{-60} \\ 570 \\ \underline{-540} \\ 300 \\ \underline{-300} \\ 0 \end{array}$$

195 hours

3. a. 75,300

b. 75,300

c. 80,000

4. $$\begin{array}{r} {}^{1} \\ 24 \\ \underline{\times\ 30} \\ 720 \end{array}$$

720 hours

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Exponent Rules: Part 2

WARM-UP

a. $a^3 \bullet a^{12}$

$= a^{3+12}$

$= a^{15}$

b. $x^{17} \div x^9$

$= x^{17-9}$

$= x^8$

PRACTICE

Students do not need to complete every problem. Problems are to be completed until five in a row (vertically, horizontally, or diagonally) are crossed off in the chart. Work for every problem is shown below.

1. $(ab)^5 = a^5b^5$

2. $\left(\frac{a}{b}\right)^{11} = \frac{a^{11}}{b^{11}}$

3. $(3^2 \bullet 2)^2 = (3^2)^2 \bullet 2^2 = 3^4 \bullet 2^2 = 81 \bullet 4 = 324$

4. $(2a^{11})^4 = 2^4(a^{11})^4 = 16a^{44}$

5. $(a^9b^4)^5 = (a^9)^5(b^4)^5 = a^{45}b^{20}$

6. $\left(\frac{2}{-3}\right)^4 = \frac{2^4}{(-3)^4} = \frac{16}{81}$

7. $\left(\frac{b^9}{f^2}\right)^6 = \frac{(b^9)^6}{(f^2)^6} = \frac{b^{54}}{f^{12}}$

8. $(bf)^3 = b^3f^3$

9. $\left(\frac{3}{2^3}\right)^4 = \frac{3^4}{(2^3)^4} = \frac{3^4}{2^{12}} = \frac{81}{4096}$

10. $(4^3 \bullet 3)^2 = (4^3)^2 \bullet 3^2 = 4^6 \bullet 3^2 = 4096 \bullet 9 = 36{,}864$

11. $(f^6)^7 = f^{42}$

12. $\left(\frac{f^5}{b^3}\right)^4 = \frac{(f^5)^4}{(b^3)^4} = \frac{f^{20}}{b^{12}}$

13. $(2^2f^3b^5)^7 = (2^2)^7(f^3)^7(b^5)^7 = 2^{14}f^{21}b^{35} = 16{,}384f^{21}b^{35}$

14. $(2^4)^5 = 2^{20} = 1{,}048{,}576$

15. $\left(\frac{a^9b^3}{f^4}\right)^5 = \frac{(a^9)^5(b^3)^5}{(f^4)^5} = \frac{a^{45}b^{15}}{f^{20}}$

16. $(a^3)^5 = a^{15}$

17. $\left(\frac{2^3}{3^2}\right)^2 = \frac{(2^3)^2}{(3^2)^2} = \frac{2^6}{3^4} = \frac{64}{81}$

18. $(6^2 \bullet 4^2)^2 = (6^2)^2 \bullet (4^2)^2 = 6^4 \bullet 4^4 = 1296 \bullet 256 = 331{,}776$

19. $\left(\frac{4a}{-5}\right)^3 = \frac{4^3a^3}{(-5)^3} = \frac{64a^3}{-125} = -\frac{64a^3}{125}$

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20. $\left(\frac{b^5 f^2}{a^7}\right)^7 = \frac{\left(b^5\right)^7 \left(f^2\right)^7}{\left(a^7\right)^7} = \frac{b^{35} f^{14}}{a^{49}}$

21. $\left(b^5 a^4\right)^3 = \left(b^5\right)^3 \left(a^4\right)^3 = b^{15} a^{12}$

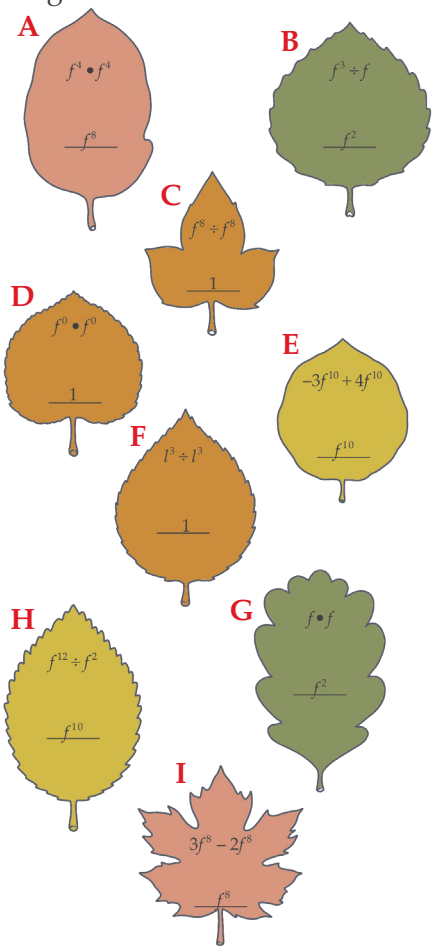
22. $\left(a^5\right)^8 = a^{40}$

23. $\left(\frac{6}{2^4}\right)^2 = \frac{6^2}{\left(2^4\right)^2} = \frac{6^2}{2^8} = \frac{36}{256} = \frac{9}{64}$

24. $\left(2^2 \cdot 3^2\right)^3 = \left(2^2\right)^3 \cdot \left(3^2\right)^3 = 2^6 \cdot 3^6 = 64 \cdot 729 = 46{,}656$

REVIEW

1. Colors will vary. Matching leaves should be in matching colors.



Detailed work for each leaf is shown below.

A $f^4 \cdot f^4 = f^{4+4} = f^8$

B $f^3 \div f = f^{3-1} = f^2$

C $f^8 \div f^8 = f^{8-8} = f^0 = 1$

D $f^0 \cdot f^0 = 1 \cdot 1 = 1$

E $-3f^{10} + 4f^{10} = 1f^{10} = f^{10}$

F $l^3 \div l^3 = l^{3-3} = l^0 = 1$

G $f \cdot f = f^{1+1} = f^2$

H $f^{12} \div f^2 = f^{12-2} = f^{10}$

I $3f^8 - 2f^8 = 1f^8 = f^8$

2. a. $-8^2 = -(8 \cdot 8) = -64$

b. $0.7^2 = 0.7 \cdot 0.7 = 0.49$

c. -1

3.
$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ -24 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

$3\frac{3}{8} = 3.375$

4. a. \$10

b. $\$4.287 \approx \$4.29$

c. \$2.82

d. \$6.10

Logic Lesson 1

Logic puzzles can be approached in many different ways. The solutions here may not represent all possible methods or answers.

Pie Partners

Amount of leftover pie:

$\frac{1}{4}+\frac{1}{3}+\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+1=3\frac{1}{2}$ pies

$3\frac{1}{2}\div 2=1\frac{3}{4}$

Each person should get a total of $1\frac{3}{4}$ pies.

More than one solution is possible.
One solution is shown below.
The total pies for each person should be $1\frac{3}{4}$ pies.

Blake: chocolate, blueberry, and apple

Nellie: pumpkin, cherry, and banana cream

Kettle Corn

More than one solution is possible. One solution is shown below.

40 pennies: 40 cents
8 nickels: 40 cents
2 dimes: 20 cents

40 cents + 40 cents + 20 cents = $1

40 pennies, 8 nickels, and 2 dimes

Helpful Hint

The size doubles every 10 minutes, so if it was 100% complete in 50 minutes, it was half the size, or 50% complete, 10 minutes before that (after 40 minutes). It was then half the size, or 25% complete, 10 minutes before that (after 30 minutes).

30 minutes

Humdinger Hayrides

If adding 5 passengers to the smaller amount and taking 5 passengers from the larger amount makes them equal, then the difference between the two amounts is 10. Taking 5 from 25 and adding it to 35 results in 20 passengers and 40 passengers. Forty is twice 20.

Ree: 35 passengers

Piper: 25 passengers

Day Debate

Since only one of the statements is true, list all of the possible days for each statement. The answer is the day of the week that is on only one list because if it appeared on more than one list, both statements would be true. The possible days for each person's statement are listed below:

Hank: Thursday, Friday, Saturday

Joe: Tuesday

Cathy: Wednesday

Olive: Monday, Tuesday, Wednesday, Friday, Saturday

Max: Monday

Monday, Tuesday, Wednesday, Friday, and Saturday are each listed twice, but Thursday is only listed once.

Thursday

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Speedy Shucking

For the first 3 minutes, Dan is the only one shucking corn. At 4 ears per minute, he shucks $4 \bullet 3 = 12$ ears of corn in 3 minutes. The two shucked 75 ears. $75 - 12 = 63$, so there are 63 ears of corn left when Delaney starts shucking. Delaney shucks 5 ears per minute, so together, Dan and Delaney shuck $5 + 4 = 9$ ears per minute. That means they'll finish shucking in $63 \div 9 = 7$ minutes. Dan will have shucked for a total of 10 minutes, so he will have shucked $10 \bullet 4 = 40$ ears of corn. Delaney will have shucked $7 \bullet 5 = 35$ ears of corn.

Dan: 40 ears of corn

Delaney: 35 ears of corn

Family Farm Assignments

Logic puzzles can be completed in different ways. Information that can be gathered from each clue is shown below.

1. Hank, Joe, and Olive do not give hayrides.

2. Joe is not assigned to the petting zoo.

3. Olive and Cathy do not close on Mondays, Wednesdays, or Fridays. That means one girl closes on Tuesdays and one on Thursdays, so none of the boys close on Tuesdays or Thursdays.

4. Since Olive and Cathy do not close on Fridays, they are not assigned to the pumpkin patch. Olive doesn't give hayrides, so she doesn't close on Thursdays. The only day left for Olive to close is Tuesdays. That means Cathy closes on Thursdays.

5. Since boys were assigned to the petting zoo and the maze, Olive was not assigned to either of those. That leaves concession stands as Olive's assignment and means that concession stands and closing on Tuesday go together.

6. If Joe doesn't close on Fridays, then he isn't assigned to the pumpkin patch. That leaves the maze as the only possible assignment for Joe.

7. The only person left who could be assigned to the pumpkin patch is Hank, and the only assignment left for Max is the petting zoo. That means Hank closes on Fridays, Max closes on Wednesdays, and Joe closes on Mondays.

		Assigned Area					Closing Day				
		Maze	Pumpkin Patch	Concession Stands	Hayrides	Petting Zoo	Monday	Tuesday	Wednesday	Thursday	Friday
Children	Hank	X	✓	X	X	X	X	X	X	X	✓
	Joe	✓	X	X	X	X	✓	X	X	X	X
	Cathy	X	X	X	✓	X	X	X	X	✓	X
	Olive	X	X	✓	X	X	X	✓	X	X	X
	Max	X	X	X	X	✓	X	X	✓	X	X
Closing Day	Monday	✓	X	X	X	X					
	Tuesday	X	X	✓	X	X					
	Wednesday	X	X	X	X	✓					
	Thursday	X	X	X	✓	X					
	Friday	X	✓	X	X	X					

a. Hank is assigned to the pumpkin patch and closes on Fridays.

b. Joe is assigned to the maze and closes on Mondays.

c. Cathy is assigned to the hayrides and closes on Thursdays.

d. Olive is assigned to the concession stands and closes on Tuesdays.

e. Max is assigned to the petting zoo and closes on Wednesdays.

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Negative Exponents

WARM-UP

$28 + 14 + 36 = 78$ children

PRACTICE

1.

Power	Fraction with Exponent	Decimal
10^{-7}	$\frac{1}{10^7}$	0.0000001
10^{-3}	$\frac{1}{10^3}$	0.001
10^{-11}	$\frac{1}{10^{11}}$	0.00000000001

2. a. 2^{-7}

$= \frac{1}{2^7}$

$= \frac{1}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$

$= \frac{1}{128}$

b. 4^{-4}

$= \frac{1}{4^4}$

$= \frac{1}{4 \cdot 4 \cdot 4 \cdot 4}$

$= \frac{1}{256}$

c. 3^{-5}

$= \frac{1}{3^5}$

$= \frac{1}{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}$

$= \frac{1}{243}$

d. $(-6)^{-3}$

$= \frac{1}{(-6)^3}$

$= \frac{1}{(-6) \cdot (-6) \cdot (-6)}$

$= -\frac{1}{216}$

e. $\frac{1}{5^{-3}}$

$= 5^3$

$= 125$

f. $\frac{1}{(-2)^{-6}}$

$= (-2)^6$

$= 64$

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3.

A $a^4 \bullet a^{-2}$	B $\frac{1}{b^{-3}}$	C $\frac{b^3}{b^3}$	a^7
1	D $a^{12} \bullet a^{-5}$	b^3	E $\frac{b^2}{b^5}$
F $b^8 \bullet b^{-5}$	G $\frac{a^8}{a^6}$	H $b \bullet b^{-4}$	I $\frac{b^{12}}{b^9}$
J $\frac{a^4}{a^{-3}}$	K a^0	L $\frac{1}{a^{-7}}$	a^2
M b^{-3}	N $\frac{1}{a^{-2}}$	O $\frac{b^{-5}}{b^{-2}}$	P $a^4 \bullet a^{-4}$

Detailed work is shown below.

A $a^4 \bullet a^{-2}$
$= a^{4+(-2)}$
$= a^2$

B $\frac{1}{b^{-3}}$
$= b^3$

C $\frac{b^3}{b^3}$
$= b^{3-3}$
$= b^0$
$= 1$

D $a^{12} \bullet a^{-5}$
$= a^{12+(-5)}$
$= a^7$

E $\frac{b^2}{b^5}$
$= b^{2-5}$
$= b^{-3}$
$= \frac{1}{b^3}$

F $b^8 \bullet b^{-5}$
$= b^{8+(-5)}$
$= b^3$

G $\frac{a^8}{a^6}$
$= a^{8-6}$
$= a^2$

H $b \bullet b^{-4}$
$= b^{1+(-4)}$
$= b^{-3}$
$= \frac{1}{b^3}$

I $\frac{b^{12}}{b^9}$
$= b^{12-9}$
$= b^3$

J $\frac{a^4}{a^{-3}}$
$= a^{4-(-3)}$
$= a^{4+3}$
$= a^7$

K $a^0 = 1$

L $\frac{1}{a^{-7}}$
$= a^7$

M b^{-3}
$= \frac{1}{b^3}$

N $\frac{1}{a^{-2}}$
$= a^2$

O $\frac{b^{-5}}{b^{-2}}$
$= b^{-5-(-2)}$
$= b^{-5+2}$
$= b^{-3}$
$= \frac{1}{b^3}$

P $a^4 \bullet a^{-4}$
$= a^{4+(-4)}$
$= a^0$
$= 1$

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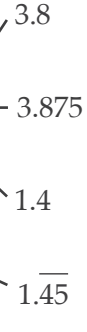
Unit 1 Assessment

1. 5043.82

2. **A** $1\frac{2}{5}$ **B** $3\frac{7}{8}$ **C** $1\frac{5}{11}$ **D** $3\frac{8}{9}$



$3.\overline{8}$ 3.875 1.4 $1.\overline{45}$

Detailed work for converting each mixed number to a decimal is shown below.

A $1\frac{2}{5} = 1\frac{4}{10} = 1.4$

B
$$\begin{array}{r} 0.875 \\ 8\overline{)7.000} \\ -\ 64 \\ \hline 60 \\ -\ 56 \\ \hline 40 \\ -\ 40 \\ \hline 0 \end{array}$$
$3\frac{7}{8} = 3.875$

C
$$\begin{array}{r} 0.4545... \\ 11\overline{)5.000...} \\ -\ 44 \\ \hline 60 \\ -\ 55 \\ \hline 50 \\ -\ 44 \\ \hline 60 \\ -\ 55 \\ \hline 50 \\ -\ 44 \\ \hline 6 \end{array}$$
$1\frac{5}{11} = 1.\overline{45}$

D
$$\begin{array}{r} 0.88... \\ 9\overline{)8.00...} \\ -\ 72 \\ \hline 80 \\ -\ 72 \\ \hline 8 \end{array}$$
$3\frac{8}{9} = 3.\overline{8}$

3. $|-15-42| = |-57| = 57$

4. a. $5\frac{7}{8} - 2\frac{1}{4}$
$= 5\frac{7}{8} - 2\frac{2}{8}$
$= 3\frac{5}{8}$

b.
$$\begin{array}{r} {\scriptstyle 1\ 1\ 1} \\ 7.485 \\ +\ 2.956 \\ \hline 10.441 \end{array}$$

c. $-\frac{8}{5} \cdot 1\frac{3}{4}$
$= -\frac{8}{5} \cdot \frac{7}{4}$
$= -\frac{14}{5} = -2\frac{4}{5}$

d.
$$\begin{array}{r} {\scriptstyle 1\quad 2} \\ 4.53 \\ \times\ \ 3.8 \\ \hline 3624 \\ +\ 13590 \\ \hline 17214 \end{array}$$
$-3.8 \cdot (-4.53) = 17.214$

e.
$$\begin{array}{r} 70 \\ 6\overline{)420} \\ -\ 42 \\ \hline 00 \\ -\ 0 \\ \hline 0 \end{array}$$

$-4.2 \div 0.06 = -70$

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f. $\dfrac{3\frac{2}{5}}{7\frac{1}{2}}$

$= 3\frac{2}{5} \div 7\frac{1}{2}$

$= \frac{17}{5} \div \frac{15}{2}$

$= \frac{17}{5} \bullet \frac{2}{15}$

$= \frac{34}{75}$

5. a. $536 + 225$

b. $52 \bullet (-43)$

6. a. $b = 123 - 54 = 69$

b. $r = 45 \div (-15) = -3$

7. a. $\left(\frac{2}{5}\right)^3$

$= \frac{2}{5} \bullet \frac{2}{5} \bullet \frac{2}{5}$

$= \frac{8}{125}$

b. -2^5

$= -(2 \bullet 2 \bullet 2 \bullet 2 \bullet 2)$

$= -32$

c. 1

8. $2|\underline{6300}$

$2|\underline{3150}$

$3|\underline{1575}$

$3|\underline{525}$

$5|\underline{175}$

$5|\underline{35}$

7

$6300 = 2^2 \bullet 3^2 \bullet 5^2 \bullet 7$

9. a. $\dfrac{2(54 - 4^3) + 18}{29 - 18 \bullet 3}$

$= \dfrac{2(54 - 64) + 18}{29 - 54}$

$= \dfrac{2(-10) + 18}{-25}$

$= \dfrac{-20 + 18}{-25}$

$= \dfrac{-2}{-25} = \dfrac{2}{25}$

b. $75 \bullet 2 \div 5 - 3^3 + (48 - 7)$

$= 75 \bullet 2 \div 5 - 3^3 + 41$

$= 75 \bullet 2 \div 5 - 27 + 41$

$= 150 \div 5 - 27 + 41$

$= 30 - 27 + 41$

$= 3 + 41$

$= 44$

10. $7q + 3q^2 - 4q + 8q^2 - 4q^2$

$= 7q - 4q + 3q^2 + 8q^2 - 4q^2$

$= 3q + 7q^2$

11. a. $3^2 \bullet 3^5 = 3^{2+5} = 3^7$

b. $a^{15} \div a^8 = a^{15-8} = a^7$

c. $7^{13} \div 7^{11} = 7^{13-11} = 7^2$

d. $z^5 \bullet z^{16} = z^{5+16} = z^{21}$

12. a. $\left(c^2 b a^3\right)^5$

$= \left(c^2\right)^5 b^5 \left(a^3\right)^5$

$= c^{10} b^5 a^{15}$

b. $\left(\frac{r^7}{ts^3}\right)^2$

$= \dfrac{\left(r^7\right)^2}{\left(ts^3\right)^2}$

$= \dfrac{r^{14}}{t^2 \left(s^3\right)^2}$

$= \dfrac{r^{14}}{t^2 s^6}$

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Modeling Real-World Situations with Equations

WARM-UP

a.
$$3x-14=7$$
$$3x-14+14=7+14$$
$$3x=21$$
$$\frac{3x}{3}=\frac{21}{3}$$
$$x=7$$

b.
$$5(x+6)=40$$
$$\frac{5(x+6)}{5}=\frac{40}{5}$$
$$x+6=8$$
$$x+6-6=8-6$$
$$x=2$$

PRACTICE

1. a. $36d-8$

b. $44d-10$

c.
$$44d-10=36d-8$$
$$44d-10-36d=36d-8-36d$$
$$8d-10=-8$$
$$8d-10+10=-8+10$$
$$8d=2$$
$$\frac{8d}{8}=\frac{2}{8}$$
$$d=\frac{1}{4}$$

$0.25 per cup

2. a. $6b+12$

b. $8b-4$

c.
$$6b+12=8b-4$$
$$6b+12-6b=8b-4-6b$$
$$12=2b-4$$
$$12+4=2b-4+4$$
$$16=2b$$
$$\frac{16}{2}=\frac{2b}{2}$$
$$8=b$$

8 bottles per package

3. a. $p+3$

b. $3(p+3)=3p+9$

c. $p+21$

d.
$$3p+9=p+21$$
$$3p+9-p=p+21-p$$
$$2p+9=21$$
$$2p+9-9=21-9$$
$$2p=12$$
$$\frac{2p}{2}=\frac{12}{2}$$
$$p=6$$

$6 per pair

4. a. $7(6p+2.5)=42p+17.5$

b.
$$55(p-0.34)+10.98$$
$$=55p-18.7+10.98$$
$$=55p-7.72$$

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c.
$$42p + 17.5 = 55p - 7.72$$
$$42p + 17.5 - 42p = 55p - 7.72 - 42p$$
$$17.5 = 13p - 7.72$$
$$17.5 + 7.72 = 13p - 7.72 + 7.72$$
$$25.22 = 13p$$
$$\frac{25.22}{13} = \frac{13p}{13}$$
$$1.94 = p$$
$$p = 1.94$$

$1.94 per pastry

REVIEW

1. $\left(4\sqrt{18} \bullet 11\right) \bullet \sqrt{2}$
$= 44\sqrt{18} \bullet \sqrt{2}$
$= 44\sqrt{18 \bullet 2}$
$= 44\sqrt{36}$
$= 44 \bullet 6$
$= 264$

2. a. $8.2 + 9.5 = 17.7$
$\left(8.2 \times 10^{-13}\right) + \left(9.5 \times 10^{-13}\right)$
$= 17.7 \times 10^{-13}$
$= 1.77 \times 10^{-12}$

b. $3.7 \times 10^{7} = 37 \times 10^{6}$
$37 - 6.01 = 30.99$
$\left(3.7 \times 10^{7}\right) - \left(6.01 \times 10^{6}\right)$
$= 30.99 \times 10^{6}$
$= 3.099 \times 10^{7}$

3. a. $\left(\frac{c^9}{5^2}\right)^4$
$= \frac{\left(c^9\right)^4}{\left(5^2\right)^4}$
$= \frac{c^{36}}{5^8}$
$= \frac{c^{36}}{390{,}625}$

b. $\left(b^{12}e^{11}\right)^{11}$
$= \left(b^{12}\right)^{11}\left(e^{11}\right)^{11}$
$= b^{132}e^{121}$

4. a. $\frac{9}{60} = \frac{3}{20}$

b. $\frac{3}{20} = \frac{15}{100}$
0.15

c. $\frac{7}{25} = \frac{28}{100}$
0.28

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The Coordinate Plane

WARM-UP

$$F = \frac{9}{5}C + 32$$

$$F - 32 = \frac{9}{5}C + 32 - 32$$

$$F - 32 = \frac{9}{5}C$$

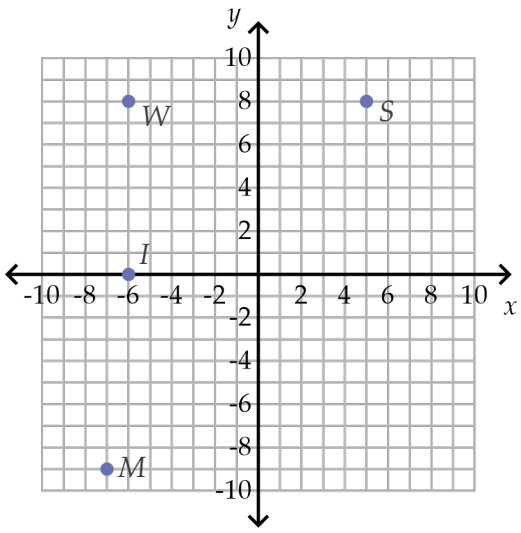
$$\frac{5}{9} \bullet (F - 32) = \frac{9}{5}C \bullet \frac{5}{9}$$

$$\frac{5}{9}(F - 32) = C$$

PRACTICE

1. F: $(-3,8)$ Quadrant: II
   R: $(0,7)$ Quadrant: N/A
   O: $(6,-4)$ Quadrant: IV
   G: $(-9,-5)$ Quadrant: III

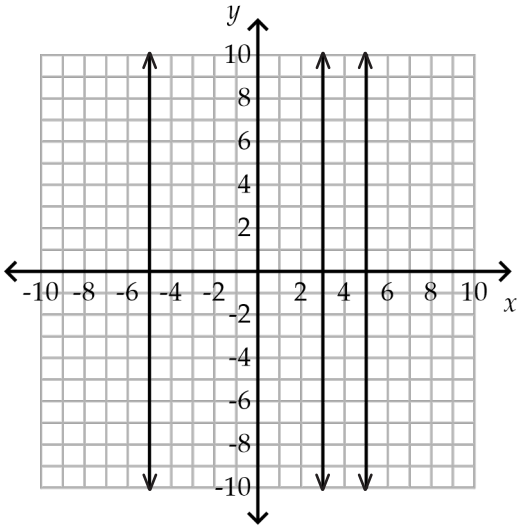
2.



3. a. x
   b. y
   c. axis
   d. $(0,0)$

4. a. $x = -3$
   b. $y = -9$

5.



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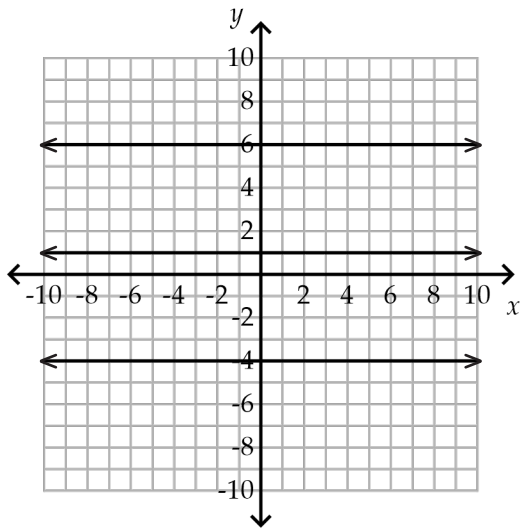
6.



REVIEW

1. $D = \frac{m}{V}$

$V \bullet D = \frac{m}{V} \bullet V$

$VD = m$

$\frac{VD}{D} = \frac{m}{D}$

$V = \frac{m}{D}$

2. $\frac{\frac{2}{3}(-9r - 60)}{2} = -2$

$2 \bullet \frac{\frac{2}{3}(-9r - 60)}{2} = -2 \bullet 2$

$\frac{2}{3}(-9r - 60) = -4$

$-6r - 40 = -4$

$-6r - 40 + 40 = -4 + 40$

$-6r = 36$

$\frac{-6r}{-6} = \frac{36}{-6}$

$r = -6$

3. a.

$2\overline{)154}$	$2\overline{)330}$
$7\overline{)77}$	$3\overline{)165}$
11	$5\overline{)55}$
	11

$154 = 2 \bullet 7 \bullet 11$

$330 = 2 \bullet 3 \bullet 5 \bullet 11$

GCF: $2 \bullet 11 = 22$

b. $\frac{154 \div 22}{330 \div 22} = \frac{7}{15}$

4. 5,879,000,000,000

5. 9 AM to 3 PM is 6 hours.

$6 \div \frac{3}{4}$

$= 6 \bullet \frac{4}{3}$

$= 8$

8 classes

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Slope-Intercept Form

WARM-UP

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{3-(-6)}{4-(-3)}$$
$$= \frac{9}{7}$$

PRACTICE

1. a. x-intercept: $(4,0)$ y-intercept: $(0,-2)$

   b. x-intercept: $(1,0)$ y-intercept: $(0,5)$

2. a. $m = 1$

   y-intercept: $\left(0,-\frac{1}{2}\right)$ $b = -\frac{1}{2}$

   b. $m = \frac{5}{4}$

   y-intercept: $(0,3)$ $b = 3$

   c. $m = 3$

   y-intercept: $(0,4)$ $b = 4$

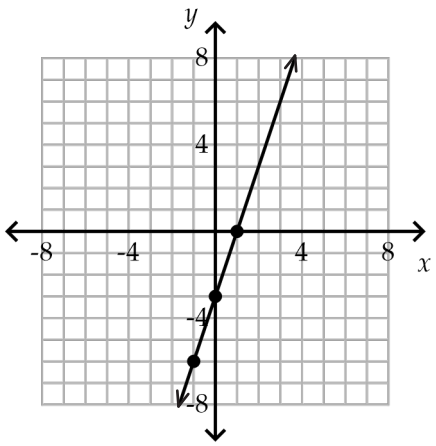
   d. $m = -5$

   y-intercept: $(0,-2)$ $b = -2$

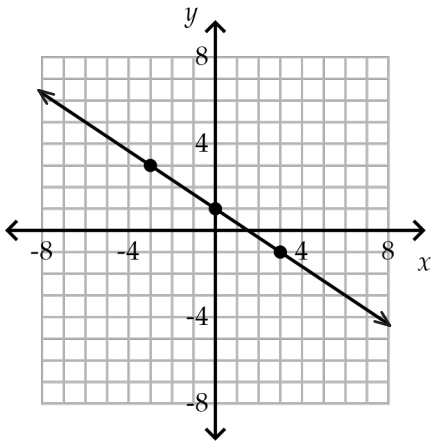
   e. $m = 1$

   y-intercept: $(0,0)$ $b = 0$

3. a.



b.



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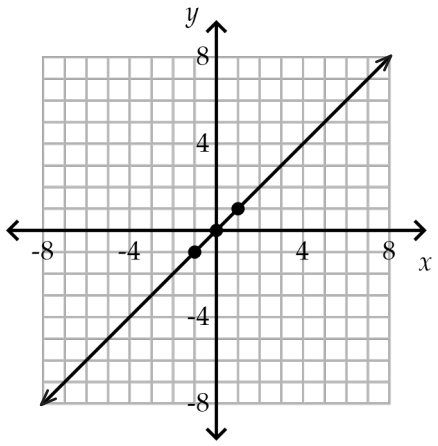
c.



4. a. $m = \frac{4}{5}$ $b = -3$

Equation: $y = \frac{4}{5}x - 3$

b. $m = -\frac{2}{7}$ $b = 0$

Equation: $y = -\frac{2}{7}x$

5. a. $y = 3x + 4$ $y = -3x - 4$

$y = -3x + 4$ $y = 3x - 4$

b $y = x + 2$ $y = -x - 2$

$y = -x + 2$ $y = x - 2$

REVIEW

1. Rule: Multiply the input by $\frac{1}{10}$, or divide the input by 10.

Equation: $y = \frac{1}{10}x$ or $y = x \div 10$

2. $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-2 - 14}{-17 - (-49)}$

$= \frac{-16}{32}$

$= -\frac{1}{2}$

3. $(4.888 \times 10^{21}) \div (9.4 \times 10^{11})$

$= \frac{4.888 \times 10^{21}}{9.4 \times 10^{11}}$

$= \frac{4.888}{9.4} \times \frac{10^{21}}{10^{11}}$

$= 0.52 \times 10^{21-11}$

$= 0.52 \times 10^{10}$

$= 5.2 \times 10^{9}$

4. a. $\sqrt{181}$ is between the whole numbers 13 and 14 but is closer to 13.

b. $\sqrt{181} \approx 13.45$

5. $0.555\ldots \approx 0.56$

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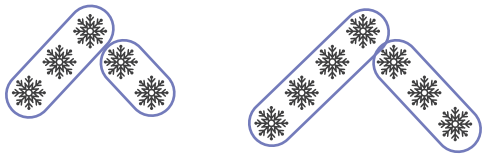
Logic Lesson 2

Logic puzzles can be approached in many different ways. The solutions here may not represent all possible methods or answers.

1. One approach is to look at the snowflakes as accumulating on the left side and right side of the pile, with the top snowflake counted on the left side.

   As shown below, the number of snowflakes on the left side (including the top snowflake) is the same as the number of seconds, and the number of snowflakes on the right side is one less than the number of seconds.

3 seconds		4 seconds	
3 on left	2 on right	4 on left	3 on right

   a. 5 seconds: $5 + 4 = 9$ snowflakes

   b. 25 seconds: $25 + 24 = 49$ snowflakes

   c. 100 seconds: $100 + 99 = 199$ snowflakes

   d. x seconds: $x + (x - 1) = 2x - 1$ snowflakes

2. Fran and her father should take path D.

   If Path A is correct, then both Mr. Jones's and Mr. Huber's statements are correct. Therefore, Path A cannot be correct.

   If Path B is correct, then both Mrs. Cunningham's and Mr. Huber's statements are correct. Therefore, Path B cannot be correct.

   If Path C is correct, then Mrs. Cunningham, Mrs. Smith, and Mr. Huber all gave correct information. Therefore, Path C cannot be correct.

   If Path D is correct, then only Mr. Jones gave correct information.

3. The man with 2 loaves of cornbread should receive 1 coin(s), and the man with 3 loaves should receive 4 coin(s).

   Each person received 5 thirds.

   Traveler A had 3 loaves. If he kept 5 thirds, he shared 4 thirds.

   Traveler B had 2 loaves. If he kept 5 thirds, he shared 1 third.

   Traveler A shared 4 times as much as Traveler B. Therefore, he should receive 4 times as many coins. $1 \bullet 4 = 4$, and $1 + 4 = 5$.

4. a. The sum of the digits 1–9 is 45. Since there are three rows, the sum of each row must be $45 \div 3$, which is 15. The same is true for each column and each diagonal. More than one solution is possible as long as the sum of each row, column, and diagonal is 15. An example solution and a possible approach are given.

2	7	6
9	5	1
4	3	8

   One approach is to list the ways to get a sum of 15 using exactly three of the numbers between 1 and 9. These include:

$1+5+9$	$1+6+8$	$2+4+9$
$2+5+8$	$2+6+7$	$3+4+8$
$3+5+7$	$4+5+6$	

   Since 5 appears in four of the ways, it must go in the middle because the middle square is included in four different sums. Since 2, 4, 6, and 8 each appear in three solutions, they must go in the corners because each corner is included in three different sums. Since 5 is already in the center, 2 and 8 must go in

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opposite corners to make 10, and 6 and 4 must also go in opposite corners to make 10. The remaining squares can be filled in with the numbers that result in a sum of 15.

b. $3 \bullet 2 - \sqrt[3]{8} = 4$

$7 - \sqrt{36} = 1$

$4 + \sqrt{9} - 2 = 5$

First Equation:

The only perfect cubes from 1 to 9 are 1 and 8. If the number 1 is under the cube root, then 2 times a number minus 1 must equal 4. Since 5 minus 4 is 1, this cannot work because 2 times any remaining number is not 5. Therefore, the number under the cube root is 8. The cube root of 8 is 2, so the number in the other box must be 3 because 6 minus 2 is 4.

Second Equation:

The only perfect square in the thirties is 36, so a 6 must go under the square root. The remaining numbers are 1, 2, 4, 5, 7, and 9. Since the square root of 36 is 6, one remaining number minus 6 has to equal another remaining number. The only remaining numbers that have a difference of 6 are 7 and 1.

Third Equation:

Now the remaining numbers are 2, 4, 5, and 9. The only perfect squares in this list are 4 and 9. Try 9 first. The square root of 9 is 3. The only way to make a true statement is to add 4 and 3 to get 7 and subtract 2 to get 5.

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Solving Equations with Radicals

WARM-UP

a. 12 b. 5 c. 20

PRACTICE

A $\sqrt{x+3}=6$ $x=33$	**B** $2\sqrt{t}=14$ $t=49$	**C** $\sqrt{r+10}=4$ $r=6$	**D** $\sqrt{a}+10=4$ $a=$ no solution
E $3\sqrt{g-1}=21$ $g=50$	**F** $5\sqrt[3]{w-7}=-5$ $w=6$	**G** $\frac{\sqrt{b+15}}{2}=4$ $b=49$	**H** $\sqrt[3]{7-4f}=-5$ $f=33$
I $\frac{5+2\sqrt{m}}{3}=1$ $m=$ no solution	**J** $\sqrt[3]{2d+5}=3$ $d=11$	**K** $5-\sqrt[3]{3-z}=7$ $z=11$	**L** $10-\sqrt{2h}=0$ $h=50$

A

$$\sqrt{x+3}=6$$
$$\left(\sqrt{x+3}\right)^2=6^2$$
$$x+3=36$$
$$x+3-3=36-3$$
$$x=33$$

Check:

$$\sqrt{33+3}\stackrel{?}{=}6$$
$$\sqrt{36}\stackrel{?}{=}6$$
$$6=6\ \checkmark$$

B

$$2\sqrt{t}=14$$
$$\frac{2\sqrt{t}}{2}=\frac{14}{2}$$
$$\sqrt{t}=7$$
$$\left(\sqrt{t}\right)^2=7^2$$
$$t=49$$

Check:

$$2\sqrt{49}\stackrel{?}{=}14$$
$$2\bullet 7\stackrel{?}{=}14$$
$$14=14\ \checkmark$$

C

$$\sqrt{r+10}=4$$
$$\left(\sqrt{r+10}\right)^2=4^2$$
$$r+10=16$$
$$r+10-10=16-10$$
$$r=6$$

Check:

$$\sqrt{6+10}\stackrel{?}{=}4$$
$$\sqrt{16}\stackrel{?}{=}4$$
$$4=4\ \checkmark$$

D

$$\sqrt{a}+10=4$$
$$\sqrt{a}+10-10=4-10$$
$$\sqrt{a}=-6$$
$$\left(\sqrt{a}\right)^2=(-6)^2$$
$$a=36$$

Check:

$$\sqrt{36}+10\stackrel{?}{=}4$$
$$6+10\stackrel{?}{=}4$$
$$16\neq 4$$

no solution

E

$$3\sqrt{g-1}=21$$
$$\frac{3\sqrt{g-1}}{3}=\frac{21}{3}$$
$$\sqrt{g-1}=7$$
$$\left(\sqrt{g-1}\right)^2=7^2$$
$$g-1=49$$
$$g-1+1=49+1$$
$$g=50$$

Check:

$$3\sqrt{50-1}\stackrel{?}{=}21$$
$$3\sqrt{49}\stackrel{?}{=}21$$
$$3\bullet 7\stackrel{?}{=}21$$
$$21=21\ \checkmark$$

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F $5\sqrt[3]{w-7}=-5$

$$\frac{5\sqrt[3]{w-7}}{5}=\frac{-5}{5}$$
$$\sqrt[3]{w-7}=-1$$
$$\left(\sqrt[3]{w-7}\right)^3=(-1)^3$$
$$w-7=-1$$
$$w-7+7=-1+7$$
$$w=6$$

Check:

$$5\sqrt[3]{6-7}\stackrel{?}{=}-5$$
$$5\sqrt[3]{-1}\stackrel{?}{=}-5$$
$$5(-1)\stackrel{?}{=}-5$$
$$-5=-5\ \checkmark$$

G $\frac{\sqrt{b+15}}{2}=4$

$$2\bullet\frac{\sqrt{b+15}}{2}=4\bullet 2$$
$$\sqrt{b+15}=8$$
$$\left(\sqrt{b+15}\right)^2=8^2$$
$$b+15=64$$
$$b+15-15=64-15$$
$$b=49$$

Check:

$$\frac{\sqrt{49+15}}{2}\stackrel{?}{=}4$$
$$\frac{\sqrt{64}}{2}\stackrel{?}{=}4$$
$$\frac{8}{2}\stackrel{?}{=}4$$
$$4=4\ \checkmark$$

H $\sqrt[3]{7-4f}=-5$

$$\left(\sqrt[3]{7-4f}\right)^3=(-5)^3$$
$$7-4f=-125$$
$$7-4f-7=-125-7$$
$$-4f=-132$$
$$\frac{-4f}{-4}=\frac{-132}{-4}$$
$$f=33$$

Check:

$$\sqrt[3]{7-4(33)}\stackrel{?}{=}-5$$
$$\sqrt[3]{7-132}\stackrel{?}{=}-5$$
$$\sqrt[3]{-125}\stackrel{?}{=}-5$$
$$-5=-5\ \checkmark$$

I $\frac{5+2\sqrt{m}}{3}=1$

$$3\bullet\frac{5+2\sqrt{m}}{3}=1\bullet 3$$
$$5+2\sqrt{m}=3$$
$$5+2\sqrt{m}-5=3-5$$
$$2\sqrt{m}=-2$$
$$\frac{2\sqrt{m}}{2}=\frac{-2}{2}$$
$$\sqrt{m}=-1$$
$$\sqrt{m}^2=(-1)^2$$
$$m=1$$

Check:

$$\frac{5+2\sqrt{1}}{3}\stackrel{?}{=}1$$
$$\frac{5+2\bullet 1}{3}\stackrel{?}{=}1$$
$$\frac{5+2}{3}\stackrel{?}{=}1$$
$$\frac{7}{3}\neq 1$$

no solution

J $\sqrt[3]{2d+5}=3$

$$\left(\sqrt[3]{2d+5}\right)^3=3^3$$
$$2d+5=27$$
$$2d+5-5=27-5$$
$$2d=22$$
$$\frac{2d}{2}=\frac{22}{2}$$
$$d=11$$

Check:

$$\sqrt[3]{2\bullet 11+5}\stackrel{?}{=}3$$
$$\sqrt[3]{22+5}\stackrel{?}{=}3$$
$$\sqrt[3]{27}\stackrel{?}{=}3$$
$$3=3\ \checkmark$$

K $5-\sqrt[3]{3-z}=7$

$$5-\sqrt[3]{3-z}-5=7-5$$
$$-\sqrt[3]{3-z}=2$$
$$\frac{-\sqrt[3]{3-z}}{-1}=\frac{2}{-1}$$
$$\sqrt[3]{3-z}=-2$$
$$(3-z)^3=(-2)^3$$
$$3-z=-8$$
$$3-z-3=-8-3$$
$$-z=-11$$
$$\frac{-z}{-1}=\frac{-11}{-1}$$
$$z=11$$

Check:

$$5-\sqrt[3]{3-11}\stackrel{?}{=}7$$
$$5-\sqrt[3]{-8}\stackrel{?}{=}7$$
$$5-(-2)\stackrel{?}{=}7$$
$$7=7\ \checkmark$$

L $10-\sqrt{2h}=0$

$$10-\sqrt{2h}-10=0-10$$
$$-\sqrt{2h}=-10$$
$$\frac{-\sqrt{2h}}{-1}=\frac{-10}{-1}$$
$$\sqrt{2h}=10$$
$$\left(\sqrt{2h}\right)^2=10^2$$
$$2h=100$$
$$\frac{2h}{2}=\frac{100}{2}$$
$$h=50$$

Check:

$$10-\sqrt{2\bullet 50}\stackrel{?}{=}0$$
$$10-\sqrt{100}\stackrel{?}{=}0$$
$$10-10\stackrel{?}{=}0$$
$$0=0\ \checkmark$$

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REVIEW

1. a. $-6x+5y=-10$

$-6x+5y+6x=-10+6x$

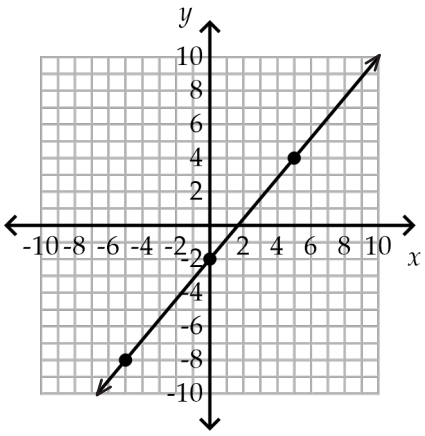
$5y=-10+6x$

$\frac{5y}{5}=\frac{-10}{5}+\frac{6x}{5}$

$y=\frac{6}{5}x-2$

Slope-intercept form: $y=\frac{6}{5}x-2$

Slope: $\frac{6}{5}$ y-intercept: $(0,-2)$



b. $m=\frac{3-(-12)}{-6-12}$

$=\frac{15}{-18}$

$=-\frac{5}{6}$

$y=mx+b$

$-12=-\frac{5}{6}(12)+b$

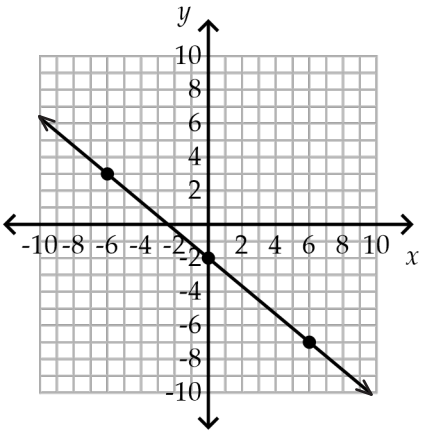
$-12=-10+b$

$-12+10=-10+b+10$

$-2=b$

Equation: $y=-\frac{5}{6}x-2$

Slope: $-\frac{5}{6}$ y-intercept: $(0,-2)$



c. The lines are perpendicular because their slopes are opposite reciprocals.

2. 1×10^{-9}

3. a. $96\div8=12$

$12\bullet5=60$

b. $108\div12=9$

$9\bullet7=63$

c. $330\div11=30$

$30\bullet5=150$

d. $250\div50=5$

$5\bullet9=45$

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Unit 2 Review

1. $2700 + 726t = 5400$

2. a.
$$\begin{aligned} 3x+1 &= 9-x \\ 3x+1+x &= 9-x+x \\ 4x+1 &= 9 \\ 4x+1-1 &= 9-1 \\ 4x &= 8 \\ \frac{4x}{4} &= \frac{8}{4} \\ x &= 2 \end{aligned}$$

b.
$$\begin{aligned} \frac{(7+19x)}{5} &= 3+4x \\ 5 \bullet \frac{(7+19x)}{5} &= (3+4x) \bullet 5 \\ 7+19x &= 15+20x \\ 7+19x-19x &= 15+20x-19x \\ 7 &= 15+x \\ 7-15 &= 15+x-15 \\ -8 &= x \end{aligned}$$

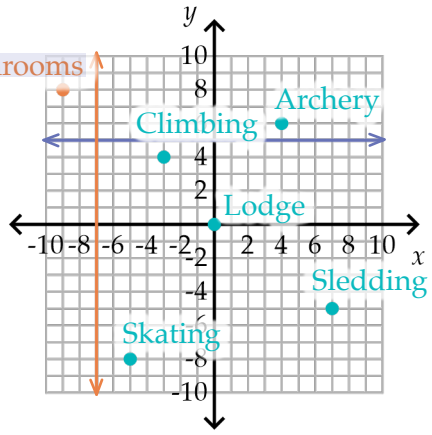
3.
$$\begin{aligned} P &= \frac{3}{2}A+5 \\ P-5 &= \frac{3}{2}A+5-5 \\ P-5 &= \frac{3}{2}A \\ \frac{2}{3} \bullet (P-5) &= \frac{3}{2}A \bullet \frac{2}{3} \\ \frac{2}{3}(P-5) &= A \end{aligned}$$

4. a. Archery: I Climbing: II
Sledding: IV Skating: III

b. Skating: $(-5,-8)$ Lodge: $(0,0)$

c. Purple: $y = 5$ Orange: $x = -7$

d. Bathrooms



5. a. Domain: $\{13, 14, 15\}$
Range: $\{\text{Cabin 1, Cabin 2, Cabin 3}\}$

b. no
The input of 13 has two outputs.

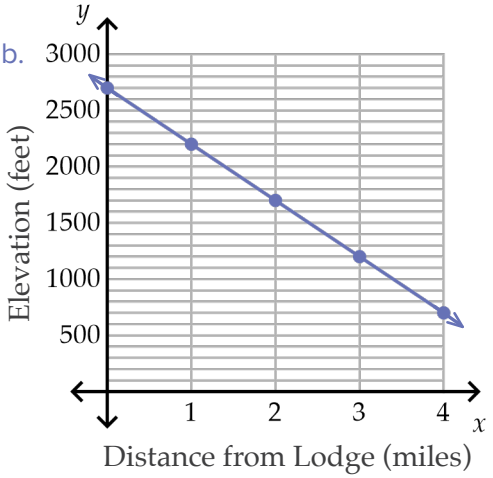
c. Independent variable: age
Dependent variable: cabin number

6. Rule: Multiply the input by $\frac{2}{5}$.
Equation: $y = \frac{2}{5}x$

7. a.

x	y	
0	2700	$-500 \bullet 0 + 2700 = 0 + 2700 = 2700$
1	2200	$-500 \bullet 1 + 2700 = -500 + 2700 = 2200$
2	1700	$-500 \bullet 2 + 2700 = -1000 + 2700 = 1700$
3	1200	$-500 \bullet 3 + 2700 = -1500 + 2700 = 1200$
4	700	$-500 \bullet 4 + 2700 = -2000 + 2700 = 700$

b.



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8. a. Function? yes
Linear? yes
Proportional? yes

b. Function? yes
Linear? no
Proportional? no

9. $y = 3x + 5$ $\quad y = x^2 - 1$ $\quad 1 + y = \frac{1}{2}x$

$2x - 5y = 4$ $\quad 2x^2 - 5y^2 = 4$ $\quad y = |x|$

10. a. $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{5-7}{7-1}$

$= \frac{-2}{6}$

$= -\frac{1}{3}$

b. $y = mx + b$

$7 = -\frac{1}{3}(1) + b$

$7 = -\frac{1}{3} + b$

$7 + \frac{1}{3} = -\frac{1}{3} + b + \frac{1}{3}$

$7\frac{1}{3} = b$

$y = -\frac{1}{3}x + 7\frac{1}{3}$

11. a. $m = \frac{\text{rise}}{\text{run}} = \frac{40}{5} = 8$

b. $(0, 30)$

c. $y = 8x + 30$

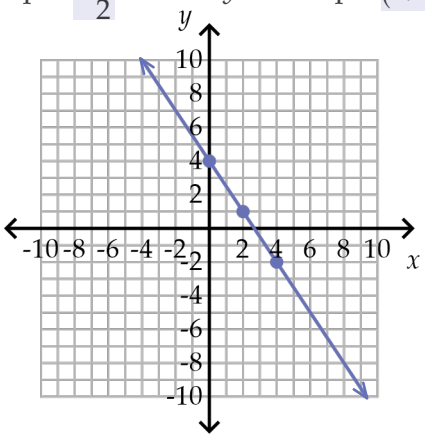
12. a. $y - (-3) = 5(x - 4)$

$y + 3 = 5(x - 4)$

b. $y - (-3) = -\frac{1}{5}(x - 4)$

$y + 3 = -\frac{1}{5}(x - 4)$

13. Slope: $-\frac{3}{2}$ $\quad y$-intercept: $(0, 4)$



14. a. $3\sqrt{a} + 1 = 13$

$3\sqrt{a} + 1 - 1 = 13 - 1$

$3\sqrt{a} = 12$

$\frac{3\sqrt{a}}{3} = \frac{12}{3}$

$\sqrt{a} = 4$

$(\sqrt{a})^2 = 4^2$

$a = 16$

Check:

$3\sqrt{16} + 1 \stackrel{?}{=} 13$

$3(4) + 1 \stackrel{?}{=} 13$

$12 + 1 \stackrel{?}{=} 13$

$13 = 13$ ✓

b. $1 - \sqrt[3]{b} = -3$

$1 - \sqrt[3]{b} - 1 = -3 - 1$

$-\sqrt[3]{b} = -4$

$\frac{-\sqrt[3]{b}}{-1} = \frac{-4}{-1}$

$\sqrt[3]{b} = 4$

$(\sqrt[3]{b})^3 = 4^3$

$b = 64$

Check:

$1 - \sqrt[3]{64} \stackrel{?}{=} -3$

$1 - 4 \stackrel{?}{=} -3$

$-3 = -3$ ✓

c. $(2c - 4)^2 = 100$

$\sqrt{(2c-4)^2} = \pm\sqrt{100}$

$2c - 4 = \pm 10$

$2c - 4 = 10$ $\quad$ $2c - 4 = -10$

$2c - 4 + 4 = 10 + 4$ $\quad$ $2c - 4 + 4 = -10 + 4$

$2c = 14$ $\quad$ $2c = -6$

$\frac{2c}{2} = \frac{14}{2}$ $\quad$ $\frac{2c}{2} = \frac{-6}{2}$

$c = 7$ $\quad$ $c = -3$

The number of snowballs Clarice made cannot be negative.

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Unit 2 Assessment

1. a. $3x+18-2x=9x-30$
$x+18=9x-30$
$x+18-x=9x-30-x$
$18=8x-30$
$18+30=8x-30+30$
$48=8x$
$\frac{48}{8}=\frac{8x}{8}$
$6=x$

b. $5(a-12)=3a+25$
$5a-60=3a+25$
$5a-60-3a=3a+25-3a$
$2a-60=25$
$2a-60+60=25+60$
$2a=85$
$\frac{2a}{2}=\frac{85}{2}$
$a=42.5$

c. $\frac{36-p}{4}=p-1$
$4\bullet\frac{36-p}{4}=(p-1)\bullet 4$
$36-p=4p-4$
$36-p+p=4p-4+p$
$36=5p-4$
$36+4=5p-4+4$
$40=5p$
$\frac{40}{5}=\frac{5p}{5}$
$8=p$

2. a. $2m+10$

b. $8m-2$

c. $2m+10=8m-2$
$2m+10-2m=8m-2-2m$
$10=6m-2$
$10+2=6m-2+2$
$12=6m$
$\frac{12}{6}=\frac{6m}{6}$
$2=m$

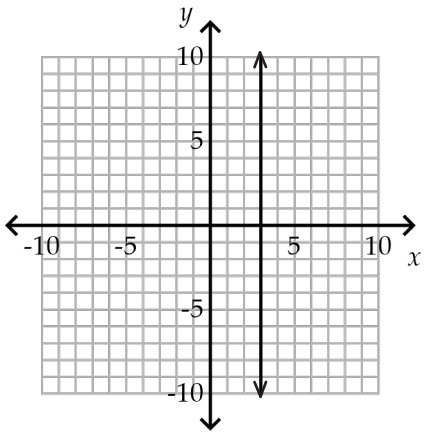
$2 per extra chore

3. a. $A=lw$
$\frac{A}{l}=\frac{lw}{l}$
$\frac{A}{l}=w$

b. $w=\frac{A}{l}$
$w=\frac{27}{9}$
$w=3$

3 ft

4.



5. $\{(0,a),(0,f),(1,b),(2,d),(3,c),(4,b),(5,f)\}$

6. Rule: Multiply the input by $\frac{1}{2}$.
Equation: $y=\frac{1}{2}x$

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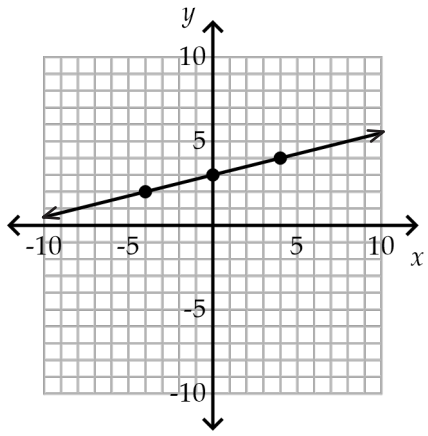
7.



linear

8. a.

Change in x	x	y	Change in y
	–5	–2.5	
+3	–2	–1	+1.5
+3	1	0.5	+1.5
+3	4	2	+1.5
+3	7	3.5	+1.5

b. yes

c. yes

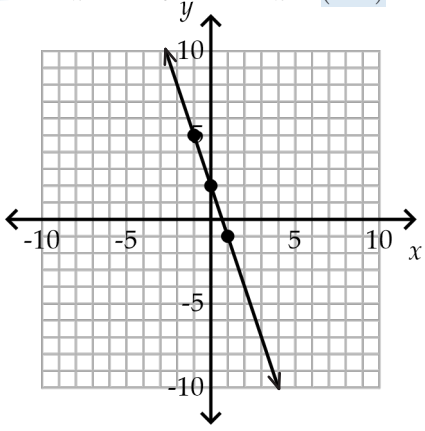
9. $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-3-11}{-5-3}$

$= \frac{-14}{-8}$

$= \frac{14}{8} = \frac{7}{4}$

10. Slope: -3 y-intercept: $(0,2)$



11. Point-slope form: $y+1=\frac{3}{2}(x-4)$

$y+1=\frac{3}{2}(x-4)$

$y+1=\frac{3}{2}x-6$

$y+1-1=\frac{3}{2}x-6-1$

$y=\frac{3}{2}x-7$

Slope-intercept form: $y=\frac{3}{2}x-7$

12. $m=\frac{-4-(-7)}{0-(-4)}=\frac{3}{4}$

$y=\frac{3}{4}x+b$

$-4=\frac{3}{4}(0)+b$

$-4=b$

$y=\frac{3}{4}x-4$

13. Cost: $\$3$/pound

Equation: $y=3x$

14. a.

$4x+3y=24$

$4x+3(0)=24$

$4x=24$

$\frac{4x}{4}=\frac{24}{4}$

$x=6$

x-intercept: $(6,0)$

$4x+3y=24$

$4(0)+3y=24$

$3y=24$

$\frac{3y}{3}=\frac{24}{3}$

$y=8$

y-intercept: $(0,8)$

b. $-x+y=-36$

$-x+0=-36$

$-x=-36$

$x=36$

x-intercept: $(36,0)$

$-x+y=-36$

$-0+y=-36$

$y=-36$

y-intercept: $(0,-36)$

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Enrichment: Collatz Conjecture

This is an enrichment lesson. Students are not expected to master content in the enrichment lessons at this level.

1. 6 is even. $6 \div 2 = 3$
   3 is odd. $3 \cdot 3 + 1 = 10$
   10 is even. $10 \div 2 = 5$
   5 is odd. $5 \cdot 3 + 1 = 16$
   16 is even. $16 \div 2 = 8$
   8 is even. $8 \div 2 = 4$
   4 is even. $4 \div 2 = 2$
   2 is even. $2 \div 2 = 1$
   1 is odd. $1 \cdot 3 + 1 = 4$

   Repeat answer: 4

2. 7 is odd. $7 \cdot 3 + 1 = 22$
   22 is even. $22 \div 2 = 11$
   11 is odd. $11 \cdot 3 + 1 = 34$
   34 is even. $34 \div 2 = 17$
   17 is odd. $17 \cdot 3 + 1 = 52$
   52 is even. $52 \div 2 = 26$
   26 is even. $26 \div 2 = 13$
   13 is odd. $13 \cdot 3 + 1 = 40$
   40 is even. $40 \div 2 = 20$
   20 is even. $20 \div 2 = 10$
   10 is even. $10 \div 2 = 5$
   5 is odd. $5 \cdot 3 + 1 = 16$
   16 is even. $16 \div 2 = 8$
   8 is even. $8 \div 2 = 4$
   4 is even. $4 \div 2 = 2$
   2 is even. $2 \div 2 = 1$
   1 is odd. $1 \cdot 3 + 1 = 4$

   Repeat answer: 4

3. 8 is even. $8 \div 2 = 4$
   4 is even. $4 \div 2 = 2$
   2 is even. $2 \div 2 = 1$
   1 is odd. $1 \cdot 3 + 1 = 4$

   Repeat answer: 4

4. The first answer to repeat in these examples is 4.

5. and 6. Answers will vary based on the starting integer chosen. Answers should follow the two rules listed at the beginning of the lesson.

7. Yes, the first repeated answer is still 4.

8. –5 is odd. $-5 \cdot 3 + 1 = -14$
   –14 is even. $-14 \div 2 = -7$
   –7 is odd. $-7 \cdot 3 + 1 = -20$
   –20 is even. $-20 \div 2 = -10$
   –10 is even. $-10 \div 2 = -5$

   Repeat answer: –5

9. –6 is even. $-6 \div 2 = -3$
   –3 is odd. $-3 \cdot 3 + 1 = -8$
   –8 is even. $-8 \div 2 = -4$
   –4 is even. $-4 \div 2 = -2$
   –2 is even. $-2 \div 2 = -1$
   –1 is odd. $-1 \cdot 3 + 1 = -2$

   Repeat answer: –2

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Simple Probability

WARM-UP

$A = P(1+r)^t$

$A = 2000(1+0.0375)^{15}$

$A = 3474.17$

$3474.17

PRACTICE

1. a. $\frac{20}{40} = 0.5 = 50\%$

   b. $\frac{10}{40} = 0.25 = 25\%$

   c. $\frac{8}{40} = 0.2 = 20\%$

   d. Note: The prime numbers from 1 to 40 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, and 37.

   $\frac{12}{40} = 0.3 = 30\%$

   e. Note: The two-digit palindromes from 1 to 40 are 11, 22, and 33.

   $\frac{3}{40} = 0.075 = 7.5\%$

2. a. $P(\text{green}) = \frac{2}{11} = 0.181818... \approx 18.18\%$

   $P(\text{yellow}) = \frac{1}{11} = 0.090909... \approx 9.09\%$

   $P(\text{red}) = \frac{3}{11} = 0.272727... \approx 27.27\%$

   $P(\text{blue}) = \frac{4}{11} = 0.363636... \approx 36.36\%$

   $P(\text{purple}) = \frac{1}{11} = 0.090909... \approx 9.09\%$

   b. green: $0.1818 \bullet 550 = 99.99 \approx 100$

   yellow: $0.0909 \bullet 550 = 49.995 \approx 50$

   red: $0.2727 \bullet 550 = 149.985 \approx 150$

   blue: $0.3636 \bullet 550 = 199.98 \approx 200$

   purple: $0.0909 \bullet 550 = 49.995 \approx 50$

   c. $P(\text{green}) = \frac{105}{550} = 0.190909... \approx 19.09\%$

   $P(\text{yellow}) = \frac{61}{550} = 0.110909... \approx 11.09\%$

   $P(\text{red}) = \frac{152}{550} = 0.276363... \approx 27.64\%$

   $P(\text{blue}) = \frac{207}{550} = 0.376363... \approx 37.64\%$

   $P(\text{purple}) = \frac{25}{550} = 0.045454... \approx 4.55\%$

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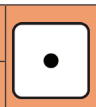
3. a.

First die → / Second die ↓						
	0	1	2	3	4	5
	1	0	1	2	3	4
	2	1	0	1	2	3
	3	2	1	0	1	2
	4	3	2	1	0	1
	5	4	3	2	1	0

b. $P(0) = \frac{6}{36} = 0.1666... \approx 16.67\%$

$P(1) = \frac{10}{36} = 0.2777... \approx 27.78\%$

$P(2) = \frac{8}{36} = 0.2222... \approx 22.22\%$

$P(3) = \frac{6}{36} = 0.1666... \approx 16.67\%$

$P(4) = \frac{4}{36} = 0.1111... \approx 11.11\%$

$P(5) = \frac{2}{36} = 0.0555... \approx 5.56\%$

c. Number of 0s: $0.1667 \bullet 400 = 66.68 \approx 67$

Number of 1s: $0.2778 \bullet 400 = 111.12 \approx 111$

Number of 2s: $0.2222 \bullet 400 = 88.88 \approx 89$

Number of 3s: $0.1667 \bullet 400 = 66.68 \approx 67$

Number of 4s: $0.1111 \bullet 400 = 44.44 \approx 44$

Number of 5s: $0.0556 \bullet 400 = 22.24 \approx 22$

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Basic Geometry Terms

WARM-UP

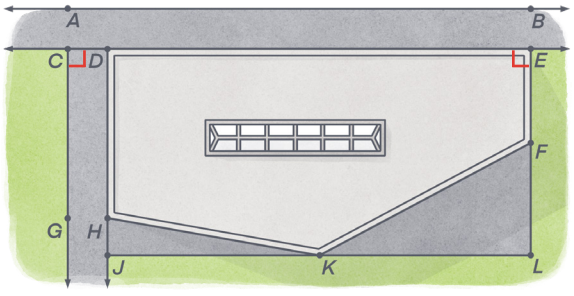
a. 6
b. 5
c. 10

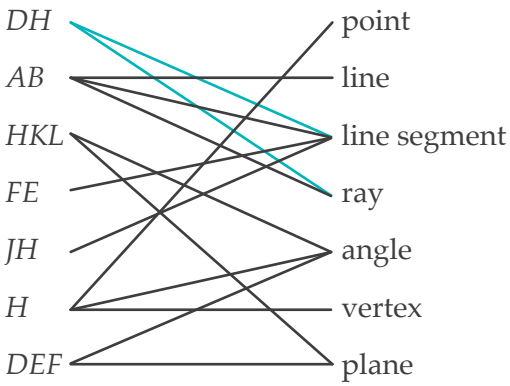
PRACTICE

1. a. $\overleftrightarrow{DE}$
   b. $\overrightarrow{CG}$
   c. K
   d. $\angle DHK$
   e. $\overline{FL}$

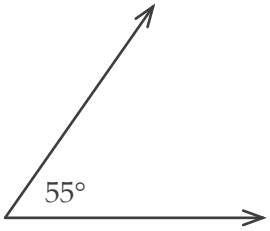
2. The exact angle measures are shown below. Answers within 1–3 degrees of the measure listed are acceptable.

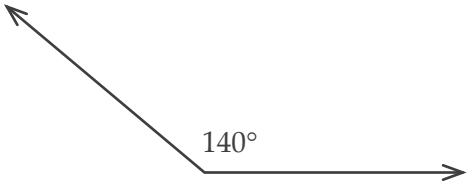
Angle	Classification	Measure
$\angle DEF$	right	90°
$\angle DHK$	obtuse	100°
$\angle HKJ$	acute	10°
$\angle GCE$	right	90°
$\angle LFK$	acute	62°

3. 

4. 

5. a. no
   b. ≅
   c. no

6. a. 

   b. 

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REVIEW

1. $x \bullet 12 = 5$

$$\frac{x \bullet 12}{12} = \frac{5}{12}$$

$$x = 0.41\overline{6} \approx 0.42$$

42%

2. a. $(0,30)$

Phillip already knew 30 digits of pi when he signed up for the competition.

b. Rise: up 80 Run: right 8

$$m = \frac{80}{8} = 10$$

Phillip learned 10 new digits per week.

c. $y = mx + b$

$y = 10x + 30$

d. $(0,5)$

Jasmine already knew five digits of pi when she signed up for the competition.

e. Rise: up 120 Run: right 8

$$m = \frac{120}{8} = 15$$

Jasmine learned 15 new digits per week.

f. $y - 80 = 15(x - 5)$

g. up to week 5

h. after week 5

i. At week 5, Phillip and Jasmine both have 80 digits of pi memorized.

j. January 17

Go back eight weeks from March 14. Two weeks (14 days) before March 14 is February 28. Four weeks (28 days) before February 28 is January 31. Two weeks (14 days) before January 31 is January 17.

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Angle Relationships and Transversals

WARM-UP

123°

PRACTICE

1. Detailed work is shown below.

Angle	Complementary	Supplementary
37°	53° **A**	143° **B**
69°	21° **C**	111° **D**
104°	none **E**	76° **F**
192°	none **G**	none **H**

A
$90° - 37° = 53°$

B
$180° - 37° = 143°$

C
$90° - 69° = 21°$

D
$180° - 69° = 111°$

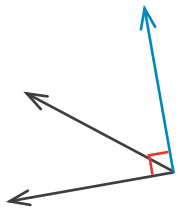
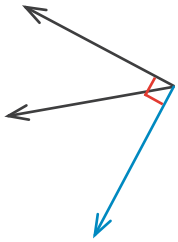
E
104° is greater than 90°.

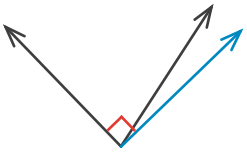
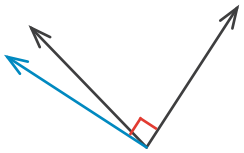
F
$180° - 104° = 76°$

G
192° is greater than 90°.

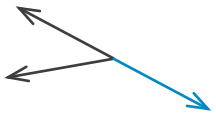
H
192° is greater than 180°.

2. The student should have drawn a ray to form a right angle with the angle given. Possible answers are shown in blue.

a.  or 

b.  or 

3. The student should have drawn a ray to form a straight line with the angle given. Possible answers are shown in blue.

a.  or 

b.  or 

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4. a. The angle that corresponds to $\angle a$ is $\angle e$, so it measures 65°.

b. $180° - 65° = 115°$

The interior angle that is adjacent to $\angle a$ is $\angle d$, and it measures 115°.

c. The angle vertical to $\angle a$ is $\angle c$, so it measures 65°.

d. An alternate interior angle for $\angle a$ is $\angle g$, so it measures 65°.

e. $180° - 65° = 115°$

The exterior angle that is adjacent to $\angle a$ is $\angle b$, and it measures 115°.

f. The angle that corresponds to $\angle b$ is $\angle f$, so it measures 115°.

g. The angle that corresponds to $\angle d$ is $\angle h$, so it measures 115°.

5.

Word	Definition
supplementary angles	two angles whose sum is 180°
adjacent angles	two angles that have the same vertex and a common side
alternate exterior angles	nonadjacent exterior angles that are located on opposite sides of the transversal
transversal	line that intersects two or more lines
corresponding angles	angles located in the same position on parallel lines when the parallel lines are cut by a transversal
vertical angles	nonadjacent angles that are opposite each other at the intersection of two lines
complementary angles	two angles whose sum is 90°
alternate interior angles	nonadjacent interior angles that are located on opposite sides of the transversal
parallel lines	lines that never intersect and are always the same distance apart

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Logic Lesson 3

Logic puzzles can be approached in many different ways. The solutions here may not represent all possible methods or answers.

1. 8 people

   There are four children (two boys and two girls), their parents, and their grandparents (their mother's mother and their father's father). The grandfather is also a father, and the grandmother is also a mother. The parents are also children: the father is a son and the mother a daughter. The grandparents are each in-laws of one of the parents, and the mother is the grandfather's daughter-in-law.

2. More than one solution is possible. One solution for each number is shown below.

   a. $13 = \frac{55+5+5}{5}$

   b. $14 = 5+5+5-\frac{5}{5}$

   c. $15 = \frac{5}{5} \bullet (5+5+5)$

3. The son is 37 years old, and his mother is 73 years old.

   Guess-and-check method:

   Start with a possible adult age for the son and reverse the digits: 24 and 42. Since 41 is not twice 23, try a new starting age close to the first one. Try 25 and 52. 51 is not double 24, but a pattern can be observed. The second digit in 41 and 51 is odd, so it cannot be twice another number. Whenever a number in the twenties is doubled and then one is subtracted, the second digit will be odd. Therefore, the son is not in his twenties.

   Start with a possible adult age for the son in the thirties and reverse the digits: 35 and 53. Since 52 is not twice 34, try a new starting age. Try 36 and 63. Since 62 is not twice 35, try a new starting age. Try 37 and 73. 72 is twice 36.

4. 1st: Somsak's family
   2nd: Malee's family
   3rd: Arthit's family
   4th: Kate's family

   If Arthit was wrong, then he was either first or last. This would mean that Malee was also wrong. The organizer said only one person was wrong, so it couldn't have been Arthit.

   If Somsak was wrong, then he did finish last. That would mean that Kate was also wrong. Only one person can be wrong, so it couldn't have been Somsak.

   If Malee was wrong, then she was not first and Arthit was not second. The other three statements could be correct, and this would be the order:

   1st: Somsak (not last)
   2nd: Malee (not first)
   3rd: Arthit (in the middle but not second)
   4th: Kate (last)

5. Logic puzzles can be completed in different ways. Information that can be gathered from each clue is shown below.

   Because there are two April 14ths, there are two possible solutions:

	Favorite Day				Favorite Event			
	April 13th	April 14th	April 14th	April 15th	Dancing	Making Garlands	Water Fight	Building Sand Pagodas
Dara (girl)	X	✓	X	X	X	X	X	✓
Mali (girl)	✓	X	X	X	✓	X	X	X
Saksit (boy)	X	X	✓	X	X	✓	X	X
Phet (boy)	X	X	X	✓	X	X	✓	X
Dancing	✓	X	X	X				
Making Garlands	X	X	✓	X				
Water Fight	X	X	X	✓				
Building Sand Pagodas	X	✓	X	X				

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	Favorite Day				Favorite Event			
	April 13th	April 14th	April 14th	April 15th	Dancing	Making Garlands	Water Fight	Building Sand Pagodas
Dara (girl)	X	X	✓	X	X	X	X	✓
Mali (girl)	✓	X	X	X	✓	X	X	X
Saksit (boy)	X	✓	X	X	X	✓	X	X
Phet (boy)	X	X	X	✓	X	X	✓	X
Dancing	✓	X	X	X				
Making Garlands	X	✓	X	X				
Water Fight	X	X	X	✓				
Building Sand Pagodas	X	X	✓	X				

Clue #1: Because the event that matches to April 14 cannot be Water Fight or Dancing, the other two events must go with April 14. Put a check mark in an April 14 column for Making Garlands and a check mark in the other April 14 column for Building Sand Pagodas.

Clue #2: Put a check mark in the April 15 column (final day of festival) for Water Fight. That means Dancing goes with April 13. Also, since a boy chose Water Fight, put an X in the row for both girls for Water Fight.

Clue #3: Since a boy chose Making Garlands, put an X in the row for both girls for Making Garlands.

Clue #4: If Dara's favorite day is not April 13 (first day), her favorite event was not Dancing. That means her favorite event was Building Sand Pagodas, which was on April 14.

From Clue #1, a boy and a girl did not choose the Water Fight and Dancing, so the other boy and girl must have chosen those events. From Clue #2, a boy chose Water Fight. Therefore, a girl chose Dancing. Since Dara did not choose Dancing, Mali chose Dancing on April 13.

Clue #5: If Phet did not choose Making Garlands, then Saksit did. Therefore, Saksit chose April 14, and Phet chose the Water Fight and April 15.

a. Dara's favorite day: April 14th
   Dara's favorite event: building sand pagodas

b. Mali's favorite day: April 13th
   Mali's favorite event: dancing

c. Saksit's favorite day: April 14th
   Saksit's favorite event: making garlands

d. Phet's favorite day: April 15th
   Phet's favorite event: water fight

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Drawings and Constructions

WARM-UP

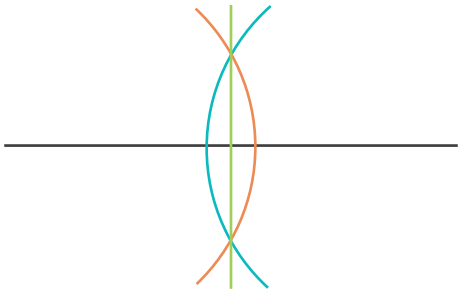
$$\frac{25}{50} = \frac{7}{3x+2}$$
$$25(3x+2) = 50 \bullet 7$$
$$75x + 50 = 350$$
$$75x + 50 - 50 = 350 - 50$$
$$75x = 300$$
$$\frac{75x}{75} = \frac{300}{75}$$
$$x = 4$$

$3(4)+2=14$

14 in

PRACTICE

1. a. Note: Arcs drawn from a compass are shown. The perpendicular bisector is the green line segment.

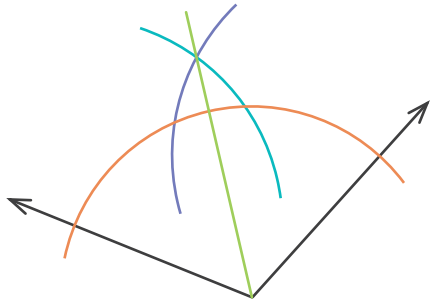


b. Left side: 3 cm Right side: 3 cm

c. The answer should be within a few degrees of 90.

90°

2. a. Note: Arcs drawn from a compass are shown. The angle bisector is the green line segment.



b. The answers should be within a few degrees of 55.

Left angle measure: 55°

Right angle measure: 55°

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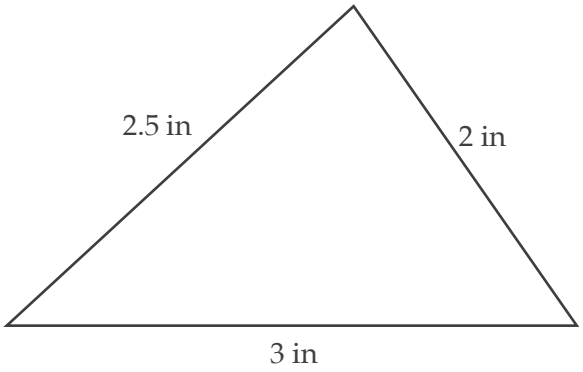
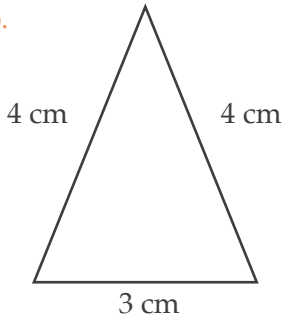
3. Note: Triangles may be oriented differently, but they should have the same side lengths.

a.

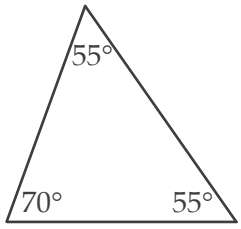


b.

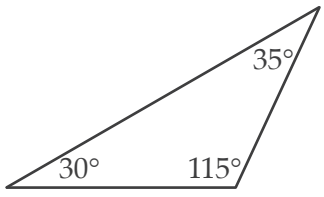


4. Note: Triangles may be oriented differently, but they should have the same angle measures.

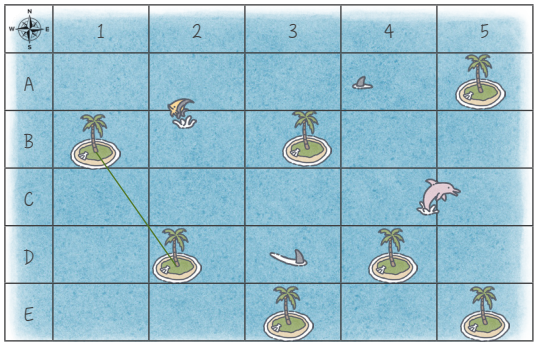
a.



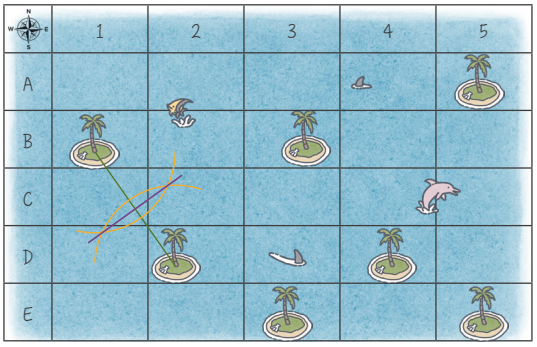
b.



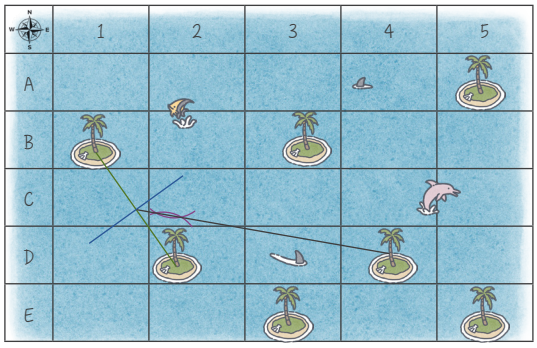
5. a.



b. Note: Arcs drawn from a compass are shown. The perpendicular bisector is the purple line segment.



c. Note: Arcs drawn from a compass are shown. The angle bisector is the black line segment.



d. D4

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Unit 3 Review

1. a. $40\% + 100\% = 140\%$

$1.4 \bullet 15 = 21$

b. $8280 - 7200 = 1080$

$$x \bullet 7200 = 1080$$
$$\frac{x \bullet 7200}{7200} = \frac{1080}{7200}$$
$$x = \frac{1080}{7200} = 0.15$$

15%

c. $100\% - 15\% = 85\%$

$$0.85 \bullet x = 78.2$$
$$\frac{0.85 \bullet x}{0.85} = \frac{78.2}{0.85}$$
$$x = 92$$

92 kg

2. a. $I = Prt$

$I = 400 \bullet 0.045 \bullet 10$

$I = 180$

$400 + 180 = 580$

$580

b. $A = P(1+r)^t$

$2{,}200{,}000 = P(1+0.07)^{30}$

$2{,}200{,}000 = P(1.07)^{30}$

$2{,}200{,}000 \approx P(7.61225504)$

$$\frac{2{,}200{,}000}{7.61225504} \approx \frac{P(7.61225504)}{7.61225504}$$
$$289{,}007 \approx P$$

$289,000

3. a. $P(\text{biologist or ecologist})$

$= P(\text{biologist}) + P(\text{ecologist})$

$$= \frac{56}{175} + \frac{62}{175}$$
$$= \frac{118}{175} \approx 0.6743$$

67.43%

b. $56 \bullet 62 \bullet 45 = 156{,}240$

156,240

c. $P(\text{no biologist, ecologist, or oceanographer})$

$= P(\text{other scientist first})$

$\bullet P(\text{other scientist second} \mid \text{other scientist first})$

$$= \frac{12}{175} \bullet \frac{11}{174}$$
$$= \frac{132}{30{,}450} \approx 0.0043$$

0.43%

4. a. $56 : 62 = 28 : 31$

b. $$\frac{45 \text{ oceanographers}}{175 \text{ scientists}} = \frac{x \text{ oceanographers}}{245 \text{ scientists}}$$
$$\frac{45}{175} = \frac{x}{245}$$
$$45 \bullet 245 = 175x$$
$$11{,}025 = 175x$$
$$\frac{11{,}025}{175} = \frac{175x}{175}$$
$$63 = x$$

5. a. $2 \text{ gal } 1 \text{ c} \bullet 24 = 48 \text{ gal } 24 \text{ c}$

$24 \text{ c} = 1 \text{ gal } 8 \text{ c} = 1 \text{ gal } 2 \text{ qt}$

$48 \text{ gal} + 1 \text{ gal } 2 \text{ qt} = 49 \text{ gal } 2 \text{ qt}$

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b. $12{,}100\ \text{yd}^2$

$$= 12{,}100\ \cancel{\text{yd}} \bullet \cancel{\text{yd}} \bullet \frac{1\ \cancel{\text{m}}}{1.1\ \cancel{\text{yd}}} \bullet \frac{1\ \cancel{\text{m}}}{1.1\ \cancel{\text{yd}}} \bullet \frac{1\ \text{km}}{1000\ \cancel{\text{m}}} \bullet \frac{1\ \text{km}}{1000\ \cancel{\text{m}}}$$

$$= \frac{12{,}100\ \text{km} \bullet \text{km}}{1.1 \bullet 1.1 \bullet 1000 \bullet 1000}$$

$$= 0.01\ \text{km}^2$$

6. a. $450 \bullet x = 1.5$

$$\frac{450 \bullet x}{450} = \frac{1.5}{450}$$

$$x = \frac{1.5}{450} = \frac{3}{900} = \frac{1}{300}$$

b. $x \bullet \frac{1}{300} = 8$

$$300 \bullet x \bullet \frac{1}{300} = 8 \bullet 300$$

$$x = 2400$$

2400 cm

c. $0.3 \bullet 300^2 = 27{,}000$

$27{,}000\ \text{m}^2$

7. $5 \bullet 250\ \text{m} = 1250\ \text{m}$

8. a. $\overleftrightarrow{AB}$

b. A

c. $\overline{AB}$

9. a. f

b. h

c. Angles a and c are supplementary, so angle c measures $180° - 73° = 107°$.

Angles c and g correspond, so angle g measures 107°.

10. a. obtuse, scalene

b. $110 + 3x + 2x - 5 = 180$

$$105 + 5x = 180$$

$$105 + 5x - 105 = 180 - 105$$

$$5x = 75$$

$$\frac{5x}{5} = \frac{75}{5}$$

$$x = 15$$

$3x = 3 \bullet 15 = 45$

$2x - 5 = 2 \bullet 15 - 5 = 30 - 5 = 25$

45° and 25°

11. a. concave

b. 5 sides $\rightarrow 3 \bullet 180° = 540°$

c. $90 + 5x + (4x - 3) + (x + 13) + 10x = 540$

$$100 + 20x = 540$$

$$100 + 20x - 100 = 540 - 100$$

$$20x = 440$$

$$\frac{20x}{20} = \frac{440}{20}$$

$$x = 22$$

A: 90°

B: $10 \bullet 22 = 220 \rightarrow 220°$

C: $22 + 13 = 35 \rightarrow 35°$

D: $4 \bullet 22 - 3 = 88 - 3 = 85 \rightarrow 85°$

E: $5 \bullet 22 = 110 \rightarrow 110°$

12. (AAA) ASA AAS

SAS (SSA) SSS

13. $\frac{30}{5} = 6$

$\frac{36}{6} = 6$

$\frac{50}{8} = 6.25$

no

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Unit 3 Assessment

1. $420{,}000 - 350{,}000 = 70{,}000$

$$x \cdot 350{,}000 = 70{,}000$$
$$\frac{x \cdot 350{,}000}{350{,}000} = \frac{70{,}000}{350{,}000}$$
$$x = 0.2$$

20% increase

2. $A = P(1+r)^t$

$$A = 450(1+0.035)^{12}$$
$$A = 450 \cdot 1.035^{12}$$
$$A \approx 450 \cdot 1.511$$
$$A \approx 679.95$$

$679.95

3. a. $120 + 80 + 100 + 150 + 50 = 500$

Pink: $\frac{120}{500} = 0.24 = 24\%$

Yellow: $\frac{80}{500} = 0.16 = 16\%$

Blue: $\frac{100}{500} = 0.2 = 20\%$

Green: $\frac{150}{500} = 0.3 = 30\%$

White: $\frac{50}{500} = 0.1 = 10\%$

b. $n = 50$

Pink: $50 \cdot 0.24 = 12$

Yellow: $50 \cdot 0.16 = 8$

Blue: $50 \cdot 0.2 = 10$

Green: $50 \cdot 0.3 = 15$

White: $50 \cdot 0.1 = 5$

4. $\frac{3}{6} \cdot \frac{1}{5} = \frac{3}{30} = \frac{1}{10}$

5. $3.78 \div 46 \approx 0.08$

$0.08 per cookie

6. $\frac{500 \text{ grams pasta}}{0.5 \text{ onions}} = \frac{x \text{ grams pasta}}{0.75 \text{ onions}}$

$$\frac{500}{0.5} = \frac{x}{0.75}$$
$$500 \cdot 0.75 = 0.5x$$
$$375 = 0.5x$$
$$\frac{375}{0.5} = \frac{0.5x}{0.5}$$
$$750 = x$$

750 grams

7. a. $4 \text{ lb} \cdot 3 = 12 \text{ lb}$

$6 \text{ oz} \cdot 3 = 18 \text{ oz}$

$18 \text{ oz} = 1 \text{ lb } 2 \text{ oz}$

$12 \text{ lb} + 1 \text{ lb } 2 \text{ oz} = 13 \text{ lb } 2 \text{ oz}$

b. $2 \text{ ft } 5 \text{ in} + 4 \text{ ft } 8 \text{ in} = 6 \text{ ft } 13 \text{ in}$

$13 \text{ in} = 1 \text{ ft } 1 \text{ in}$

$6 \text{ ft} + 1 \text{ ft } 1 \text{ in} = 7 \text{ ft } 1 \text{ in}$

8. $\frac{40 \text{ lb}}{1} \cdot \frac{1 \text{ kg}}{2.2 \text{ lb}} = \frac{40 \text{ kg}}{2.2} \approx 18.18 \text{ kg}$

9. $320 \text{ ft} : 4 \text{ in} \rightarrow 80 \text{ ft} : 1 \text{ in}$

10. acute

11. $\angle a$ and $\angle f$

$\angle b$ and $\angle e$

$\angle c$ and $\angle h$

$\angle d$ and $\angle g$

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12. $6x+1+8x+1+11x+3=180$

$25x+5=180$

$25x+5-5=180-5$

$25x=175$

$\frac{25x}{25}=\frac{175}{25}$

$x=7$

$m\angle P=(11(7)+3)°$

$m\angle P=(77+3)°$

$m\angle P=80°$

13.

Shape	Reason
quadrilateral	It has four sides.
parallelogram	It has two pairs of parallel sides.
rhombus	All sides are congruent.

14. side-angle-side

15. $\frac{JG}{NK}=\frac{GH}{KL}$

$\frac{2.5}{1}=\frac{4.5}{p}$

$2.5p=4.5$

$\frac{2.5p}{2.5}=\frac{4.5}{2.5}$

$p=1.8$

16. Perimeter of semicircle:

$C=\pi d$

$C=25\pi$

$C\approx 78.54$

Divide C by 2: $78.54\div 2=39.27$

Hypotenuse of triangle:

$25^2+25^2=c^2$

$625+625=c^2$

$1250=c^2$

$\sqrt{1250}=\sqrt{c^2}$

$c\approx 35.36$

$P=39.27\text{ ft}+35.36\text{ ft}+25\text{ ft}=99.63\text{ ft}$

17. a. $\frac{22.5°}{360°}=\frac{1}{16}$

$C=2\pi r$

$C=2\pi\bullet 6.5=13\pi$

$13\pi\bullet\frac{1}{16}=\frac{13}{16}\pi\approx 2.55$

2.55 cm

b. $A=\pi r^2$

$A=\pi(6.5)^2$

$A=42.25\pi$

$42.25\pi\bullet\frac{1}{16}=\frac{42.25}{16}\pi\approx 8.30$

8.30 cm^2

18. $A_{tri}=\frac{1}{2}bh$

$A_{tri}=\frac{1}{2}(0.5)(0.75)$

$A_{tri}\approx 0.188$

0.188 in^2

$A_{semi}=\frac{1}{2}\pi r^2$

$A_{semi}=\frac{1}{2}\pi(0.25)^2$

$A_{semi}=\frac{1}{2}\pi(0.0625)$

$A_{semi}=0.03125\pi$

$A_{semi}\approx 0.098$

0.098 in^2

Total Area: $6(0.188\text{ in}^2+0.098\text{ in}^2)=1.716\text{ in}^2$

19. $SA_{cyl}=\pi r^2+2\pi rh$

$SA_{cyl}=\pi(15)^2+2\pi(15)(25)$

$SA_{cyl}=225\pi+750\pi$

$SA_{cyl}=975\pi$

$SA_{cyl}\approx 3063.05$

3063.05 ft^2

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Enrichment: Tessellations

This is an enrichment lesson. Students are not expected to master content in the enrichment lessons at this level.

Shape (number of sides)	Interior Angle Measure	360° ÷ Interior Angle Measure	Does the shape tessellate?
Equilateral Triangle (3)	$\frac{180°(n-2)}{n}$ $=\frac{180°(3-2)}{3}$ $=60°$	$360° \div 60° = 6$	yes
Square (4)	$\frac{180°(n-2)}{n}$ $=\frac{180°(4-2)}{4}$ $=90°$	$360° \div 90° = 4$	yes
Regular Pentagon (5)	$\frac{180°(n-2)}{n}$ $=\frac{180°(5-2)}{5}$ $=108°$	$360° \div 108° = 3.\overline{3}$	no
Regular Hexagon (6)	$\frac{180°(n-2)}{n}$ $=\frac{180°(6-2)}{6}$ $=120°$	$360° \div 120° = 3$	yes
Regular Heptagon (7)	$\frac{180°(n-2)}{n}$ $=\frac{180°(7-2)}{7}$ $\approx 128.57°$	$360° \div 128.57° \approx 2.8$	no
Regular Octagon (8)	$\frac{180°(n-2)}{n}$ $=\frac{180°(8-2)}{8}$ $=135°$	$360° \div 135° = 2.\overline{6}$	no

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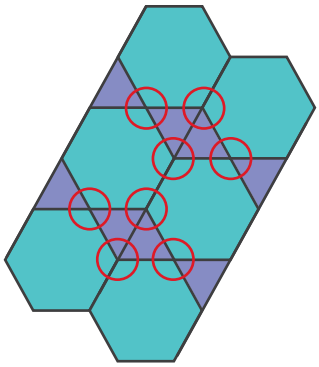
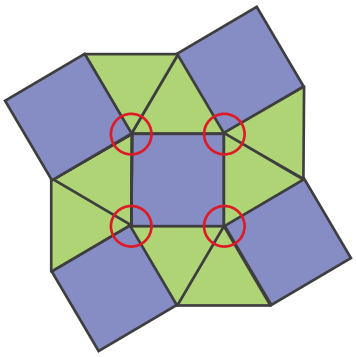


Regular hexagon: 120°

Equilateral triangle: 60°

Interior angle sum at intersection:
60° + 60° + 120° + 120° = 360°



Square: 90°

Equilateral triangle: 60°

Interior angle sum at intersection:
90° + 90° + 60° + 60° + 60° = 360°

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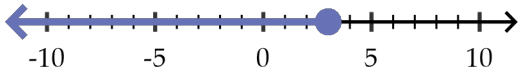
Graphing Linear Inequalities

WARM-UP

$$4t - 8 \geq 9t - 23$$
$$4t - 8 + 8 \geq 9t - 23 + 8$$
$$4t \geq 9t - 15$$
$$4t - 9t \geq 9t - 15 - 9t$$
$$-5t \geq -15$$
$$\frac{-5t}{-5} \geq \frac{-15}{-5}$$
$$t \leq 3$$



PRACTICE

1. Red:

Slope: 2

y-intercept: $(0,-6)$

Test $(0,0)$:

$y > 2x - 6$

$0 \stackrel{?}{>} 2 \bullet 0 - 6$

$0 \stackrel{?}{>} 0 - 6$

$0 \stackrel{?}{>} -6$ ✓

Yellow:

Slope: $\frac{3}{4}$

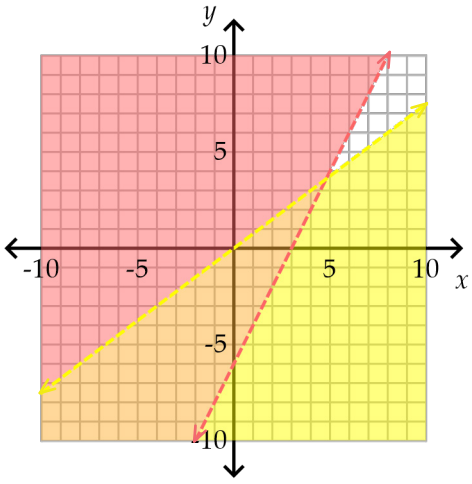
y-intercept: $(0,0)$

Test $(0,1)$:

$y < \frac{3}{4}x$

$1 \stackrel{?}{<} \frac{3}{4}(0)$

$1 \stackrel{?}{<} 0$ ✗



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2.

Ordered pair	$y > 2x - 6$	$y < \frac{3}{4}x$
$(0,4)$	$y > 2x - 6$ $4 \stackrel{?}{>} 2(0) - 6$ $4 \stackrel{?}{>} -6$ ✓ yes	$y < \frac{3}{4}x$ $4 \stackrel{?}{<} \frac{3}{4}(0)$ $4 \stackrel{?}{<} 0$ ✗ no
$(0,-4)$	$y > 2x - 6$ $-4 \stackrel{?}{>} 2(0) - 6$ $-4 \stackrel{?}{>} -6$ ✓ yes	$y < \frac{3}{4}x$ $-4 \stackrel{?}{<} \frac{3}{4}(0)$ $-4 \stackrel{?}{<} 0$ ✓ yes
$(4,0)$	$y > 2x - 6$ $0 \stackrel{?}{>} 2(4) - 6$ $0 \stackrel{?}{>} 2$ ✗ no	$y < \frac{3}{4}x$ $0 \stackrel{?}{<} \frac{3}{4}(4)$ $0 \stackrel{?}{<} 3$ ✓ yes
$(-4,0)$	$y > 2x - 6$ $0 \stackrel{?}{>} 2(-4) - 6$ $0 \stackrel{?}{>} -14$ ✓ yes	$y < \frac{3}{4}x$ $0 \stackrel{?}{<} \frac{3}{4}(-4)$ $0 \stackrel{?}{<} -3$ ✗ no

3. Blue:

Slope: $-\frac{5}{3}$

y-intercept: $(0,3)$

Test $(0,0)$:

$y \leq -\frac{5}{3}x + 3$

$0 \stackrel{?}{\leq} -\frac{5}{3}(0) + 3$

$0 \stackrel{?}{\leq} 3$ ✓

Yellow:

Slope: $\frac{1}{2}$

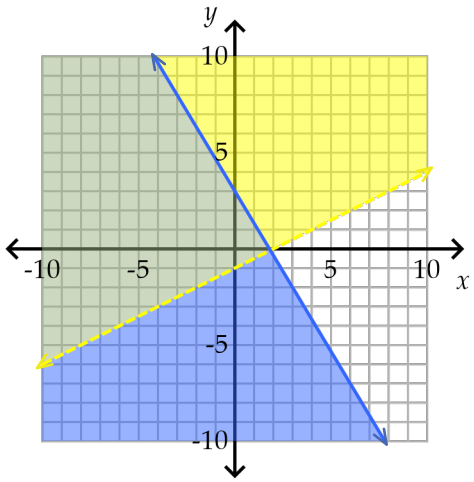
y-intercept: $(0,-1)$

Test $(0,0)$:

$y > \frac{1}{2}x - 1$

$0 \stackrel{?}{>} \frac{1}{2}(0) - 1$

$0 \stackrel{?}{>} -1$ ✓



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Solving Systems by Substitution

WARM-UP

a. 1700

b. 2.31

c. 0.0005487

PRACTICE

Note: There are many ways to solve systems of equations. Work may vary, but the solutions should be the same.

1. Solve for x:

$$\begin{aligned} -x-3y &= -1 \\ -x-3y+3y &= -1+3y \\ -x &= -1+3y \\ \frac{-x}{-1} &= \frac{-1}{-1}+\frac{3y}{-1} \\ x &= 1-3y \end{aligned}$$

Substitution:

$$\begin{aligned} 3x+4y &= -2 \\ 3(1-3y)+4y &= -2 \\ 3-9y+4y &= -2 \\ 3-5y &= -2 \\ 3-5y-3 &= -2-3 \\ -5y &= -5 \\ \frac{-5y}{-5} &= \frac{-5}{-5} \\ y &= 1 \end{aligned}$$

$$\begin{aligned} x &= 1-3y \\ x &= 1-3(1) \\ x &= -2 \end{aligned}$$

Solution: $(-2,1)$

2. Solve for y:

$$\begin{aligned} 4x+2y &= 22 \\ 4x+2y-4x &= 22-4x \\ 2y &= 22-4x \\ \frac{2y}{2} &= \frac{22}{2}-\frac{4x}{2} \\ y &= 11-2x \end{aligned}$$

Substitution:

$$\begin{aligned} 2x-3y &= -9 \\ 2x-3(11-2x) &= -9 \\ 2x-33+6x &= -9 \\ 8x-33 &= -9 \\ 8x-33+33 &= -9+33 \\ 8x &= 24 \\ \frac{8x}{8} &= \frac{24}{8} \\ x &= 3 \end{aligned}$$

$$\begin{aligned} y &= 11-2x \\ y &= 11-2(3) \\ y &= 5 \end{aligned}$$

Solution: $(3,5)$

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Substitution:

$$\begin{aligned} -4x+6y &= -52 \\ -4\left(13+\frac{3}{2}y\right)+6y &= -52 \\ -52-6y+6y &= -52 \\ -52 &= -52 \end{aligned}$$

Solution: infinitely many solutions

7. Solve for y:

$$\begin{aligned} -5x-y &= 0 \\ -5x-y+5x &= 0+5x \\ -y &= 5x \\ \frac{-y}{-1} &= \frac{5x}{-1} \\ y &= -5x \end{aligned}$$

Substitution:

$$\begin{aligned} x+y &= 0 \\ x+(-5x) &= 0 \\ -4x &= 0 \\ \frac{-4x}{-4} &= \frac{0}{-4} \\ x &= 0 \end{aligned}$$

$$\begin{aligned} y &= -5x \\ y &= -5(0) \\ y &= 0 \end{aligned}$$

Solution: $(0,0)$

8. Solve for y:

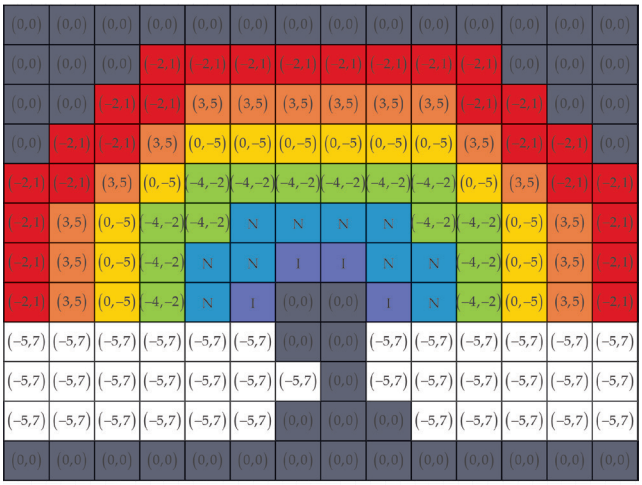
$$\begin{aligned} 2x+y &= -3 \\ 2x+y-2x &= -3-2x \\ y &= -3-2x \end{aligned}$$

Substitution:

$$\begin{aligned} 3x+2y &= -1 \\ 3x+2(-3-2x) &= -1 \\ 3x-6-4x &= -1 \\ -x-6 &= -1 \\ -x-6+6 &= -1+6 \\ -x &= 5 \\ \frac{-x}{-1} &= \frac{5}{-1} \\ x &= -5 \end{aligned}$$

$$\begin{aligned} y &= -3-2x \\ y &= -3-2(-5) \\ y &= 7 \end{aligned}$$

Solution: $(-5,7)$



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Rotations and Symmetry

WARM-UP

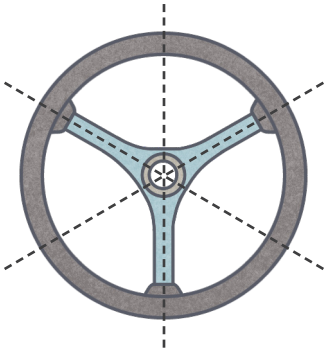
a. 12.5%

b. 0.08%

c. 145%

PRACTICE

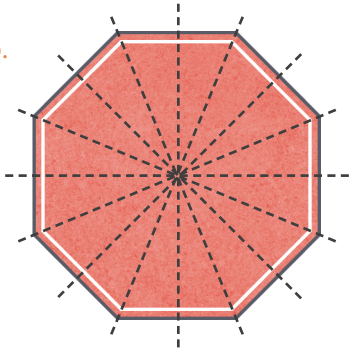
1. a.



Lines of symmetry: 3

Order: 3

b.



Lines of symmetry: 8

Order: 8

c.

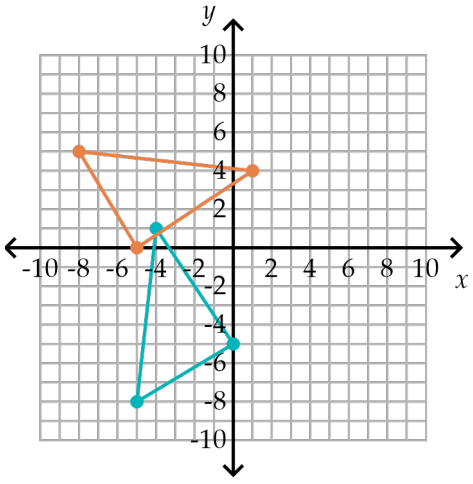


Lines of symmetry: 0

Order: 1

2. a. Rule: $(x,y) \rightarrow (-y,x)$

Rotate 90° Counterclockwise			
Preimage	$(-8,5)$	$(-5,0)$	$(1,4)$
Image	$(-5,-8)$	$(0,-5)$	$(-4,1)$



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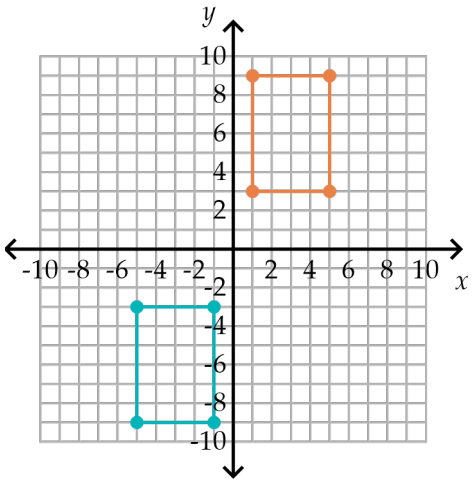
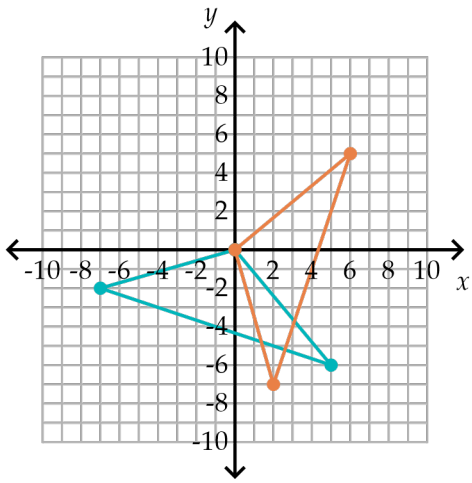
b. Rule: $(x,y) \rightarrow (-x,-y)$

Rotate 180° Clockwise				
Preimage	$(1,9)$	$(5,9)$	$(5,3)$	$(1,3)$
Image	$(-1,-9)$	$(-5,-9)$	$(-5,-3)$	$(-1,-3)$



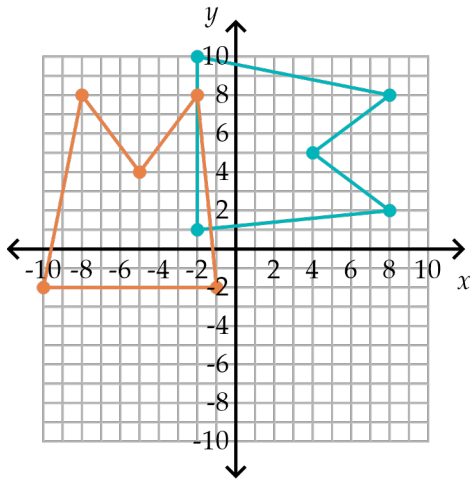
c. Rule: $(x,y) \rightarrow (y,-x)$

Rotate 90° Clockwise			
Preimage	$(6,5)$	$(0,0)$	$(2,-7)$
Image	$(5,-6)$	$(0,0)$	$(-7,-2)$



d. Rule: $(x,y) \rightarrow (y,-x)$

Rotate 270° Counterclockwise					
Preimage	$(-2,8)$	$(-5,4)$	$(-8,8)$	$(-10,-2)$	$(-1,-2)$
Image	$(8,2)$	$(4,5)$	$(8,8)$	$(-2,10)$	$(-2,1)$



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